東京大学 物性研究所
THE INSTITUTE FOR SOLID STATE PHYSICS
THE UNIVERSITY OF TOKYO
PRINCETON UNIVERSITY

Entanglement Phase Transition Induced by the Non-Hermitian Skin Effect

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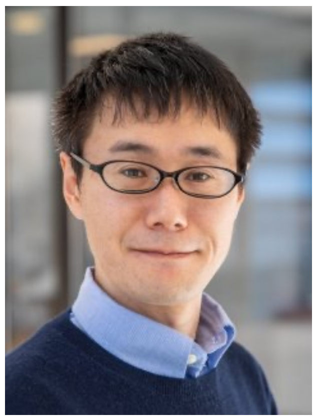
Phys. Rex. X 13, 021007 (2023)

Collaborators



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Outline

1. Introduction

2. Entanglement suppression induced by the non-Hermitian skin effect

3. Entanglement phase transition induced by the non-Hermitian skin effect

4. Purification induced by the non-Hermitian skin effect in Lindbladians

Non-Hermitian physics

Despite the enormous success, the existing framework of topological phases is **confined to Hermitian systems at equilibrium**.

→ **Richer properties appear in non-Hermitian systems!**

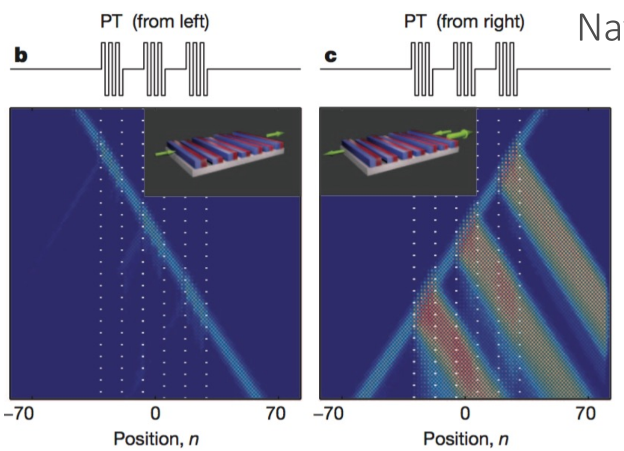
☆ Non-Hermiticity arises from **dissipation**, i.e., exchanges of energy or particles with an environment.

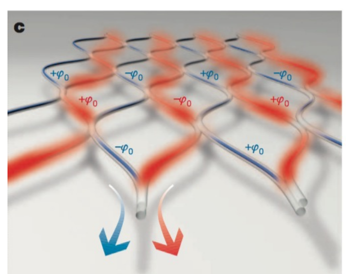
El-Ganainy *et al.*, Nat. Phys. **14**, 11 (2018)

- **Photonic lattices with gain/loss**

Unidirectional light transport

Regensburger *et al.*, Nature **488**, 167 (2012)

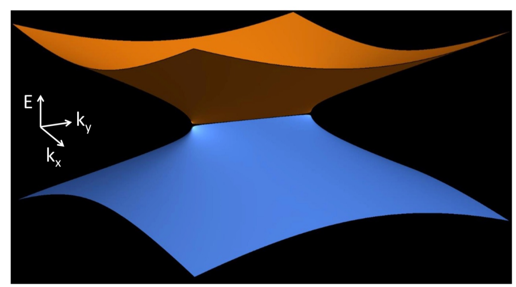


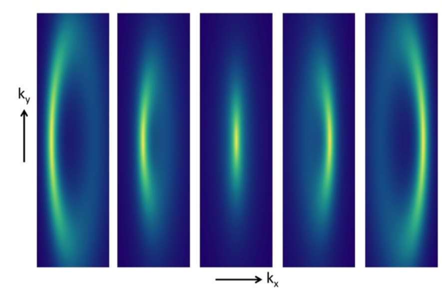


- **Finite-lifetime quasiparticles**

Bulk Fermi arc due to non-Hermitian self-energy

Kozii & Fu, arXiv: 1708.05841

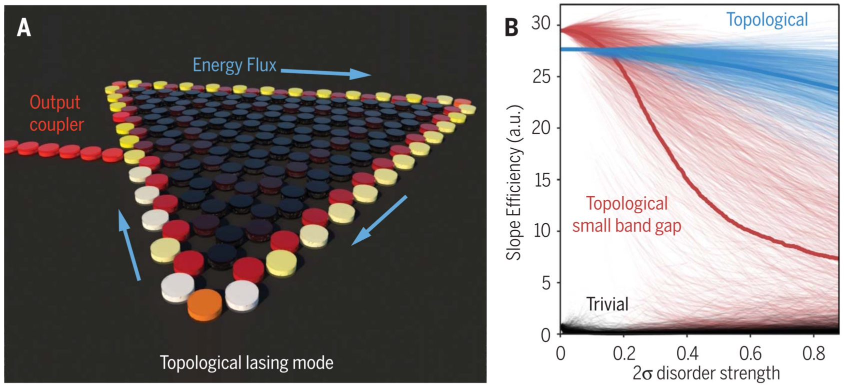




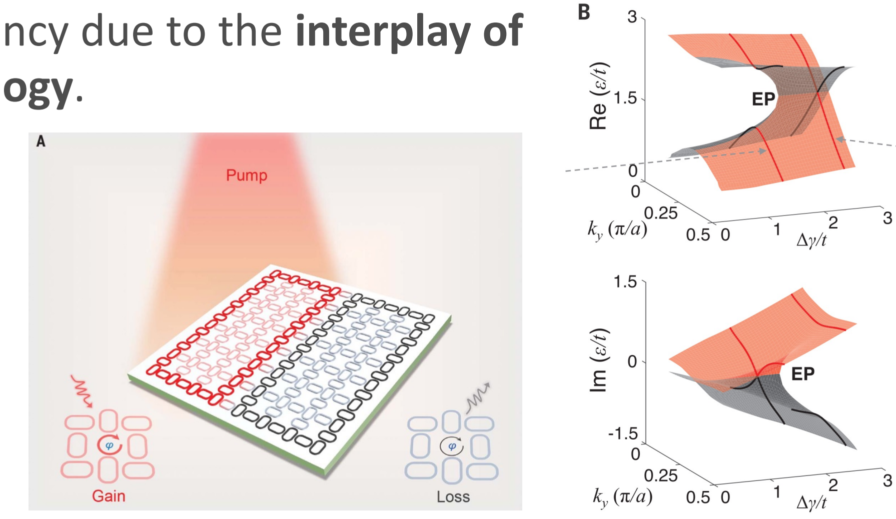
Non-Hermitian topological systems

- **Topological laser**

New laser with high efficiency due to the **interplay of non-Hermiticity and topology**.



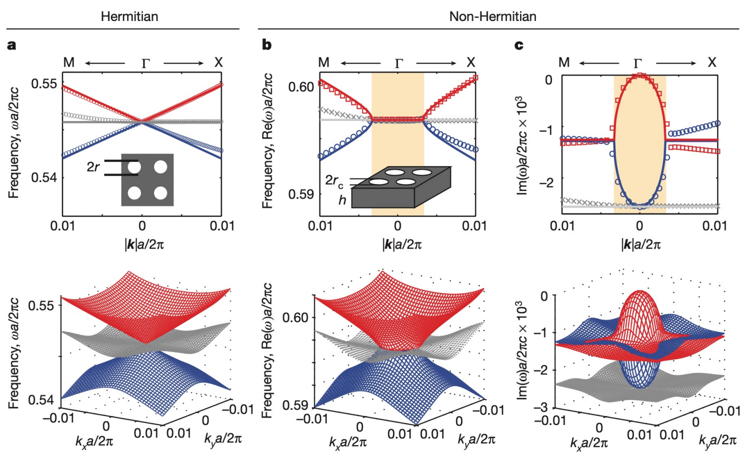
Bandres *et al.*, Science **359**, eaar4005 (2018)



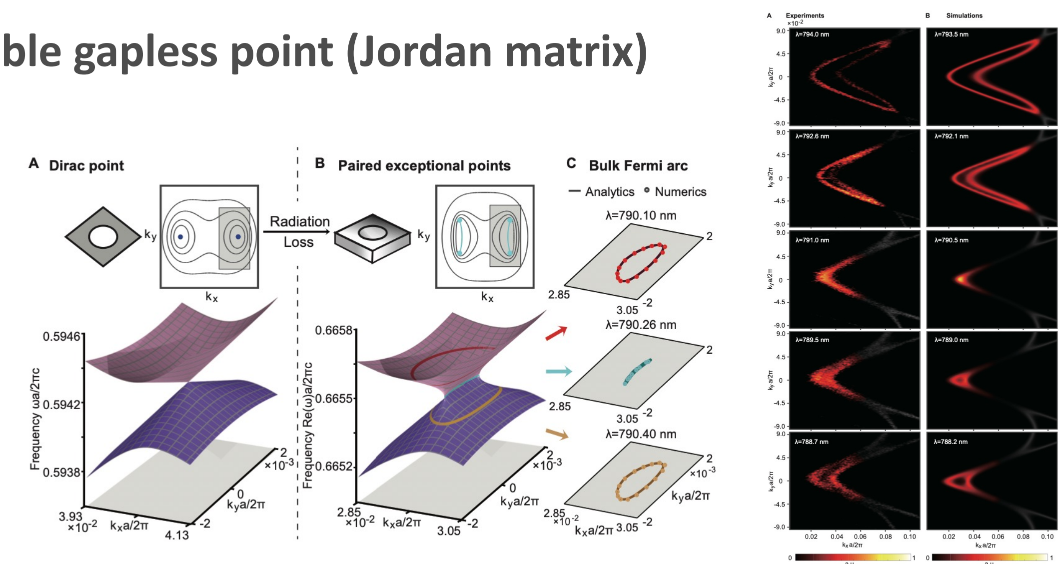
Zhao *et al.*, Science **365**, 1163 (2019)

- **Exceptional point**

Nondiagonalizable gapless point (Jordan matrix)



Zhen *et al.*, Nature **525**, 354 (2015)



Zhou *et al.*, Science **359**, 1009 (2018)

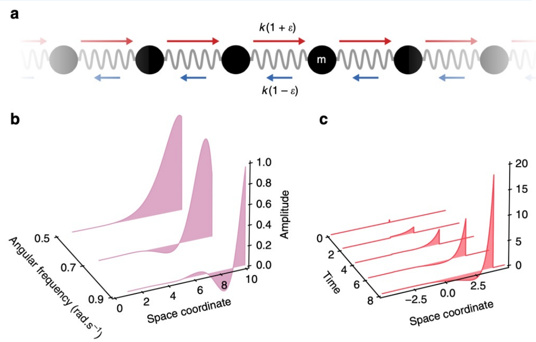
Non-Hermitian skin effect

☆ Non-Hermitian skin effect

Lee, PRL **116**, 133903 (2016); Yao & Wang, PRL **121**, 086803 (2018); Kunst *et al.*, PRL **121**, 026808 (2018)

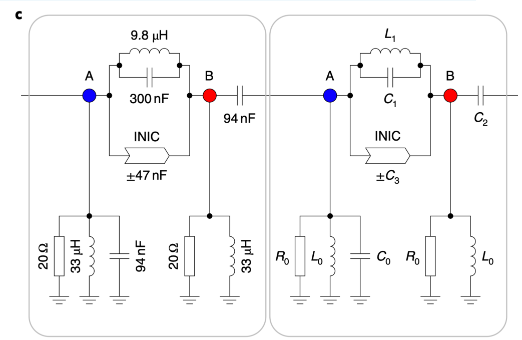
Localization of an extensive number of eigenmodes due to non-Hermitian topology

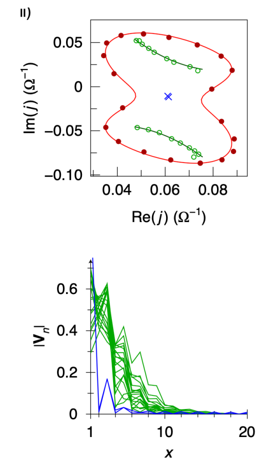
- **Mechanical metamaterials**



Brandenbourger *et al.*, Nat. Commun. **10**, 4608 (2019)

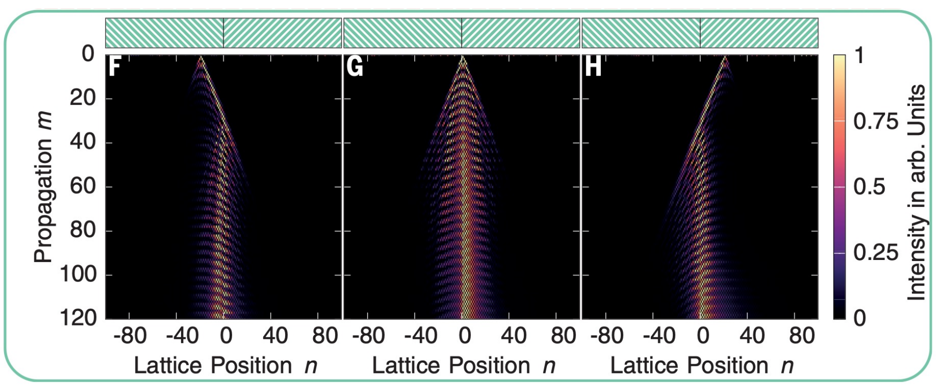
- **Electrical circuits**





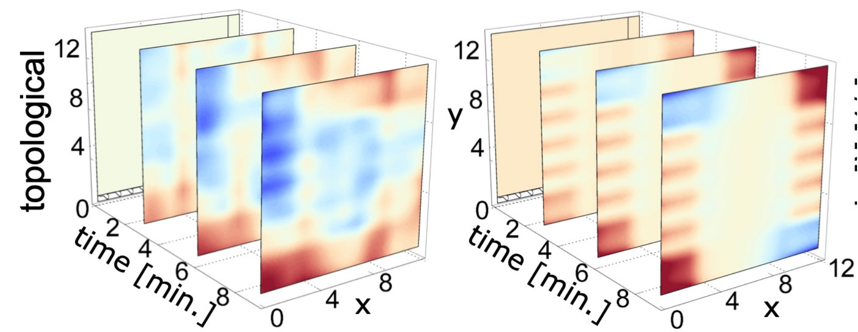
Helbig *et al.*, Nat. Phys. **16**, 747 (2020)

- **Photonic lattice**



Weidemann *et al.*, Science **368**, 311 (2020)

- **Active matter**



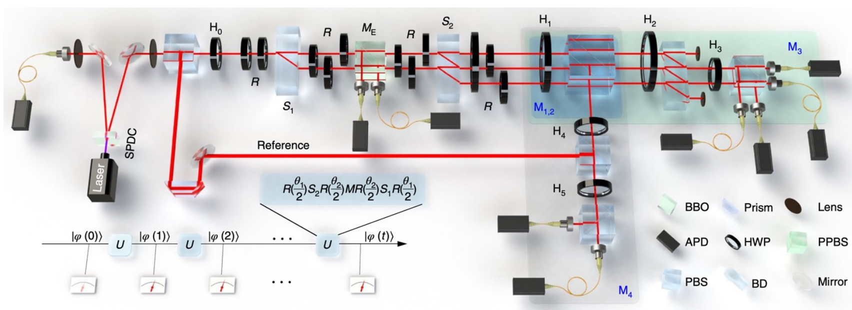
Palacios *et al.*, Nat. Commun. **12**, 4691 (2021)

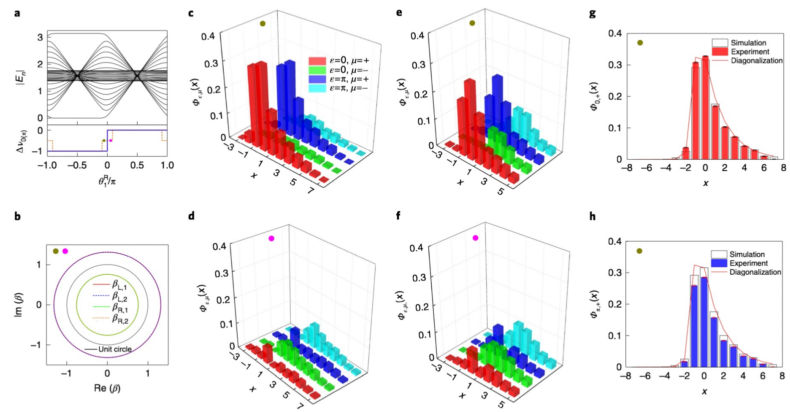
Skin effect in quantum physics

Skin effect has been observed also in recent quantum experiments.

- **Quantum walk (single photons)**

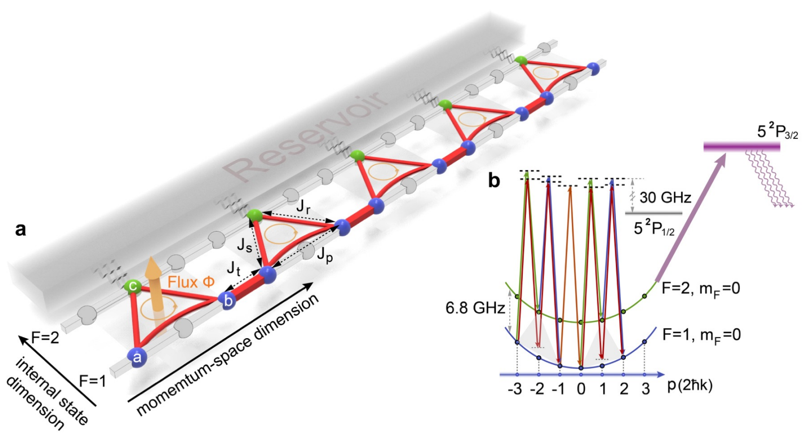
Xiao *et al.*, Nat. Phys. **16**, 761 (2020)

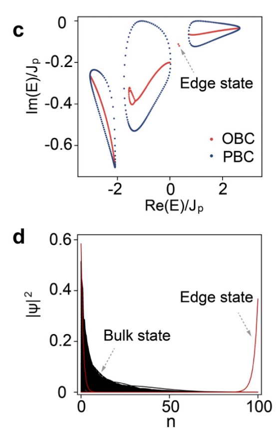


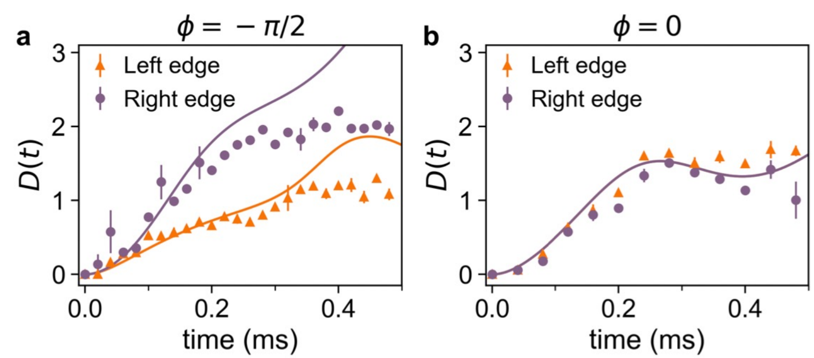


- **Ultracold atoms**

Liang *et al.*, PRL **129**, 070401 (2022)







Skin effect in quantum physics

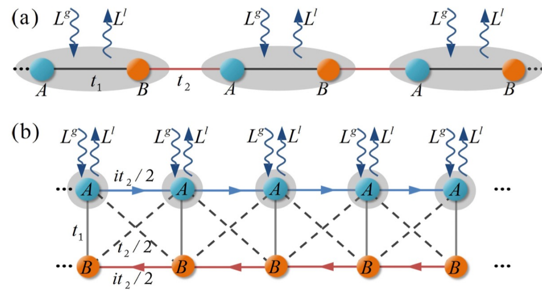
Non-Hermitian topology is relevant even to more generic open quantum systems that are not characterized by Hamiltonians.

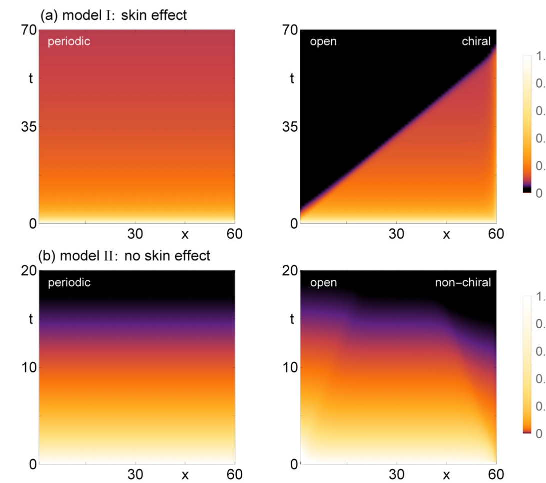
Song, Yao & Wang, PRL **123**, 170401 (2019)

- **Master equation**

$$d\hat{\rho}/dt = \underline{\mathcal{L}}\hat{\rho}$$

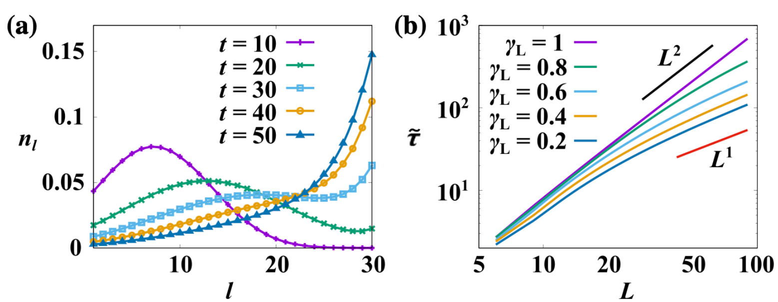
Non-Hermitian superoperator (Liouvillian)

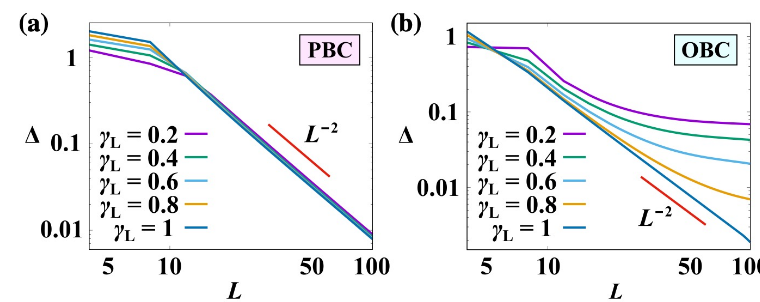




Chiral damping due to the skin effect

Slowdown of relaxation due to the skin effect





Haga *et al.*, PRL **127**, 070402 (2021); Mori *et al.*, PRL **125**, 230604 (2020)

Motivation and Results

Despite the significance of the skin effect for non-Hermitian topology, its impact on genuine quantum nature has remained unclear.

We show that the **non-Hermitian skin effect** plays an important role in the **entanglement dynamics of open quantum systems**.

(1) The skin effect **prohibits the entanglement propagation and thermalization**, leading to the area law for the steady state.

(2) The skin effect even triggers a **new type of entanglement phase transition** characterized by a new nonunitary CFT.

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Complex-energy gaps (1)

An “**energy gap**” is needed to define a topological phase.

However, a non-Hermitian extension of an **“energy gap”** is nontrivial since the **spectrum is complex**.

Hermitian

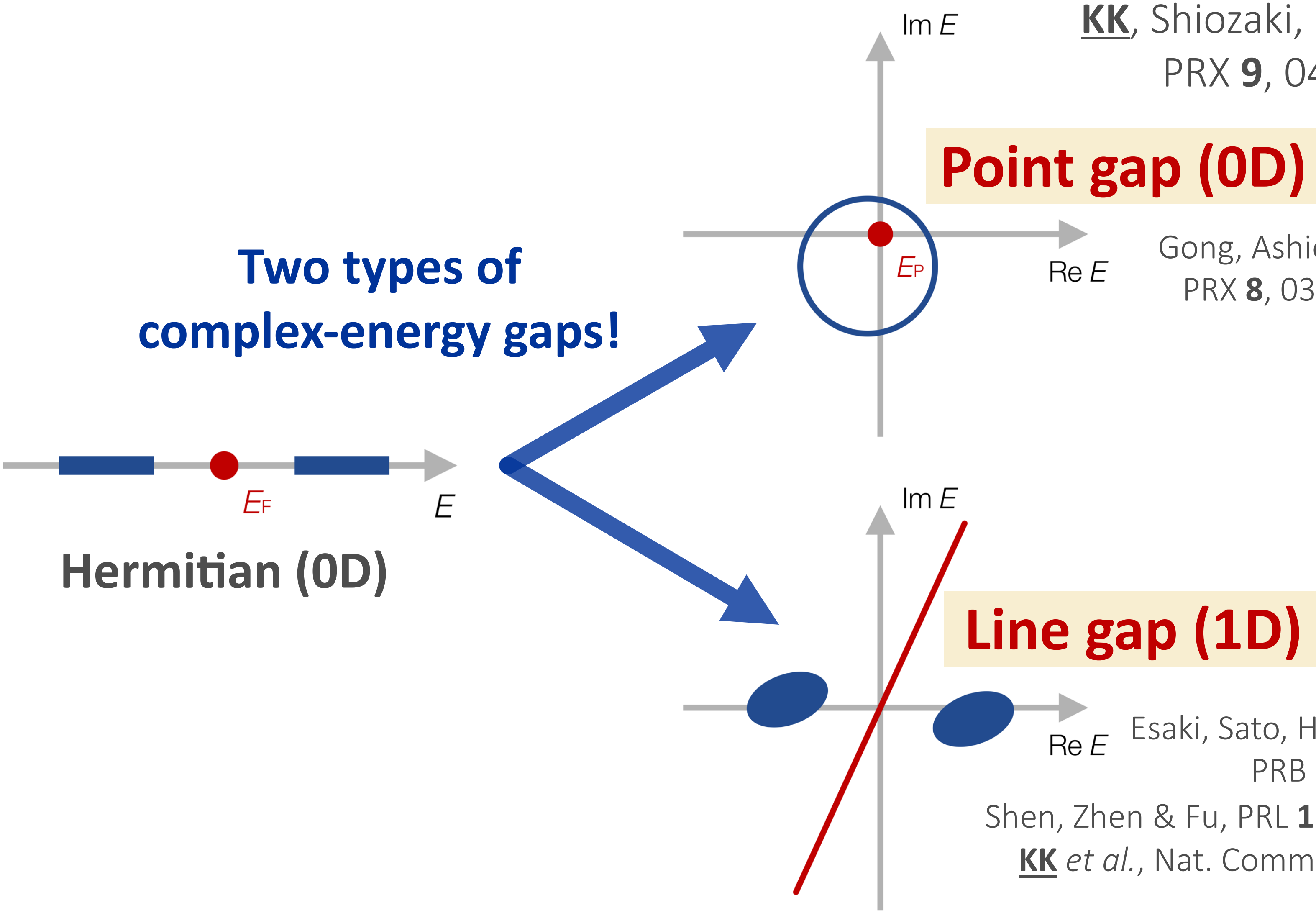
Energy gap in Hermitian systems:

- Energy regions where states are **forbidden to be present**.
- They **should be point-like (0D)** since the **real spectrum is 1D**.

→ Since the **complex spectrum is 2D (real and imaginary)**, such vacant regions can be **either 0D or 1D**!

Complex-energy gaps (2)

KK, Shiozaki, Ueda & Sato, PRX **9**, 041015 (2019)



Gong, Ashida, **KK** *et al.*, PRX **8**, 031079 (2018)

Esaki, Sato, Hasebe & Kohmoto, PRB **84**, 205128 (2011)

Shen, Zhen & Fu, PRL **120**, 146402 (2018)

KK *et al.*, Nat. Commun. **10**, 297 (2019)

NOTE: The definition that should be adopted depends on the individual physical situations that we are interested in.

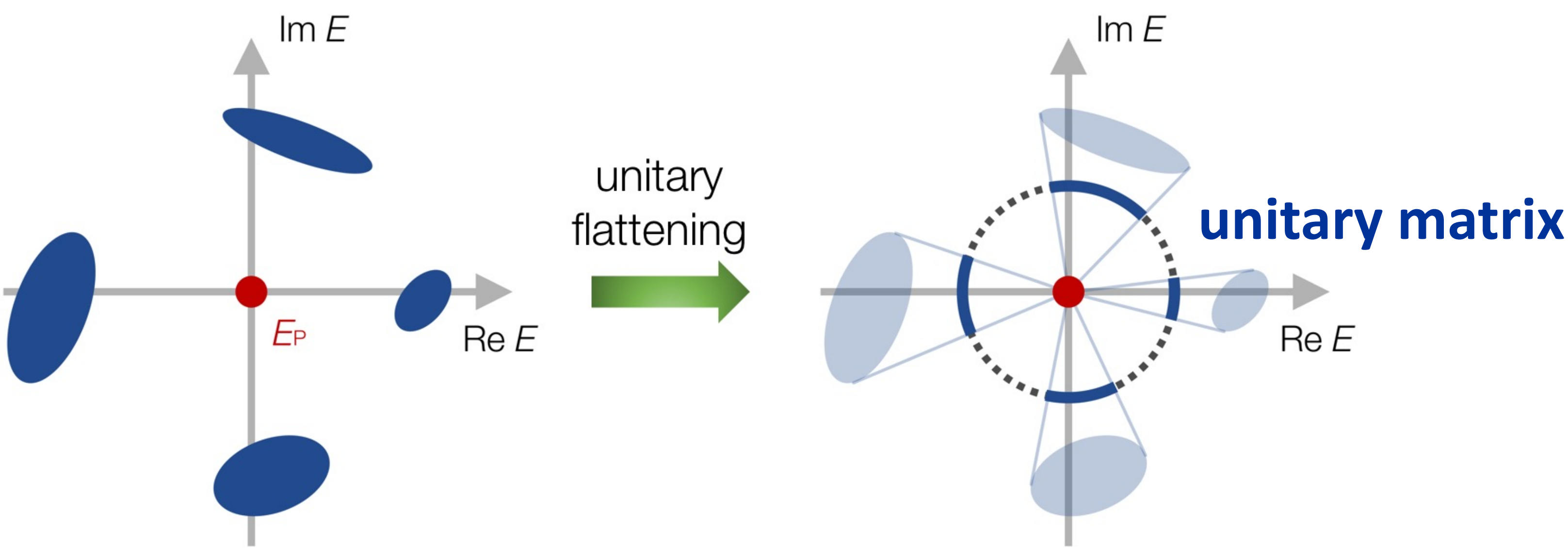
Point gap: unitary flattening

Unitary flattening for a point gap:

A non-Hermitian Hamiltonian with a **point gap** can be continuously deformed into a **unitary matrix** while having symmetry and the point gap.

non-Hermitian H ⟷ unitary U ⟷ **Hermitian** $\begin{pmatrix} 0 & U \\ U^\dagger & 0 \end{pmatrix}$

Classification is well established!



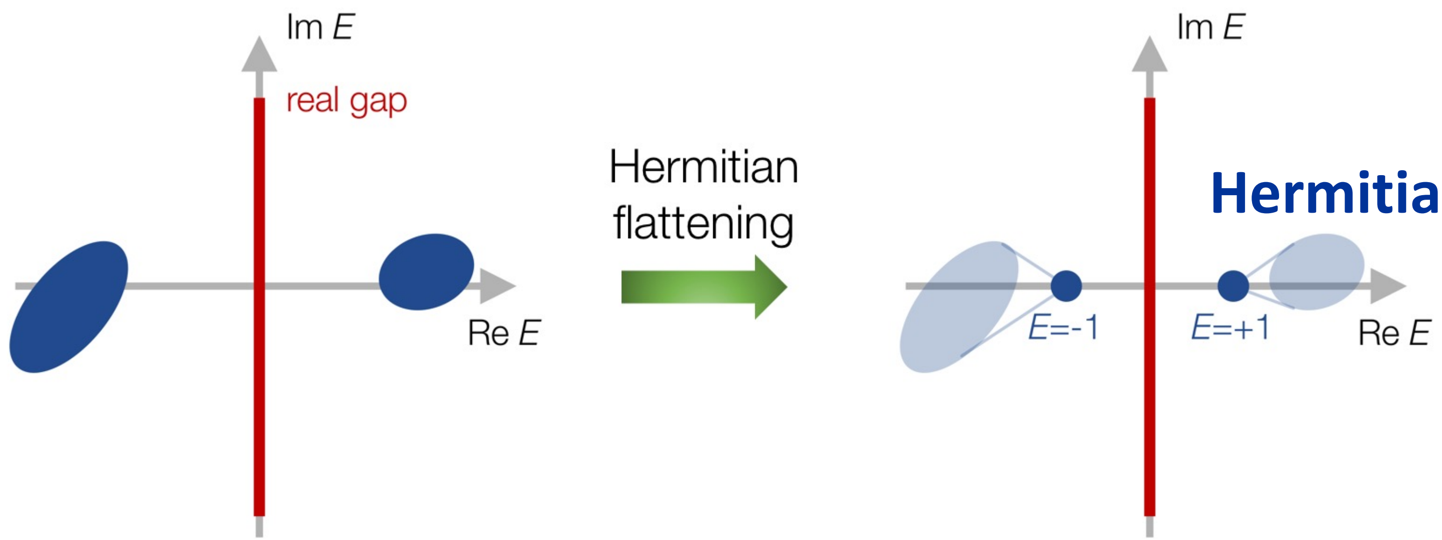
Line gap: Hermitian flattening

Hermitian flattening for a line gap:

A non-Hermitian Hamiltonian with a **line gap** can be continuously deformed into a **Hermitian matrix** while having symmetry and the line gap.

non-Hermitian H ⟷ **Hermitian** $\tilde{H}$

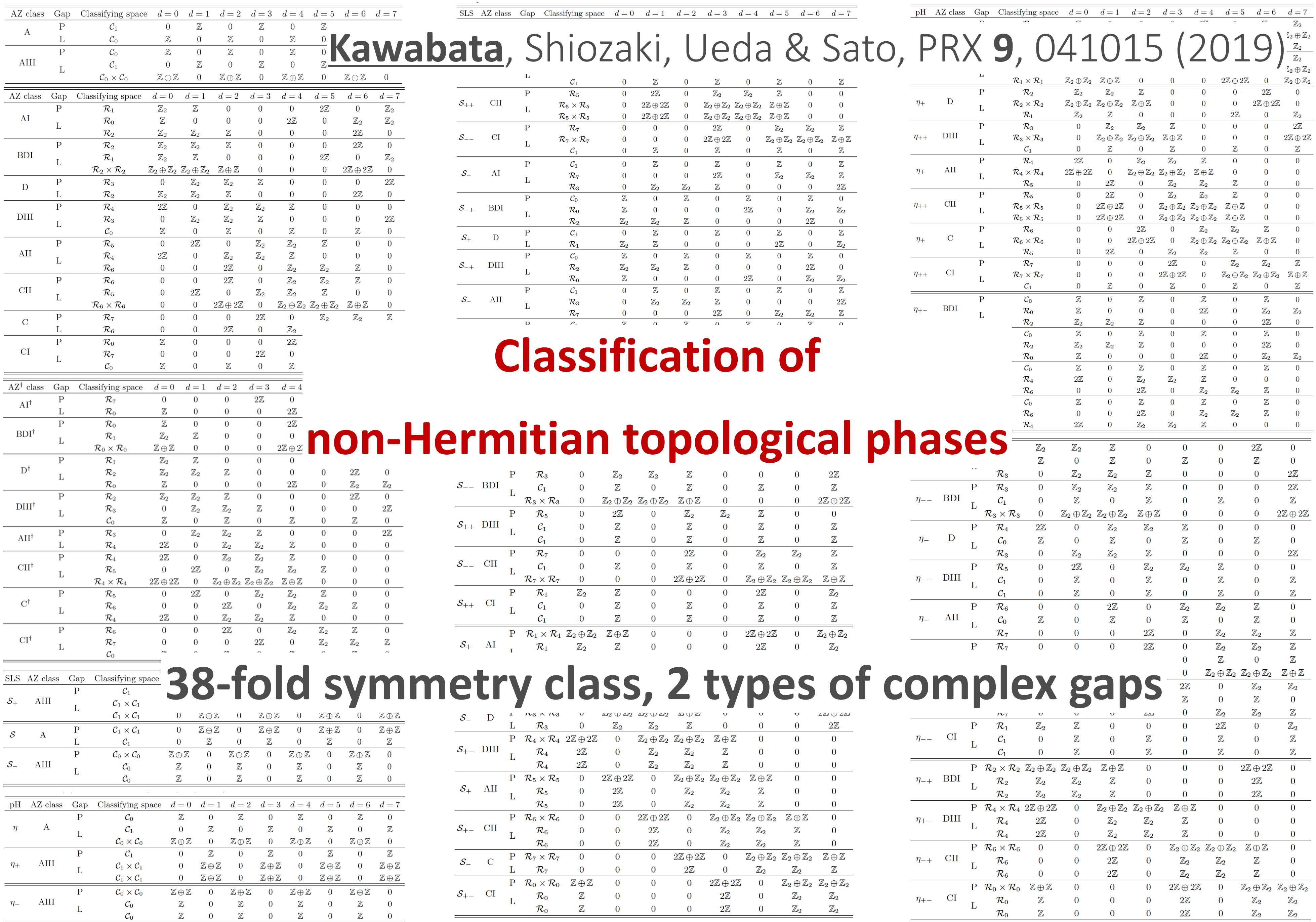
Classification is well established!



Kawabata, Shiozaki, Ueda & Sato, PRX **9**, 041015 (2019)

Classification of non-Hermitian topological phases

38-fold symmetry class, 2 types of complex gaps



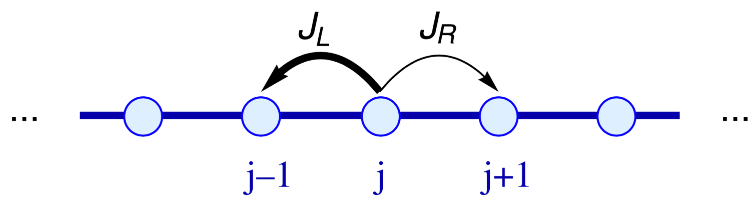
Intrinsic non-Hermitian topology

Line-gap topology: stability of Hermitian topology against non-Hermiticity

⟷ **Point-gap topology: intrinsic non-Hermitian topology**

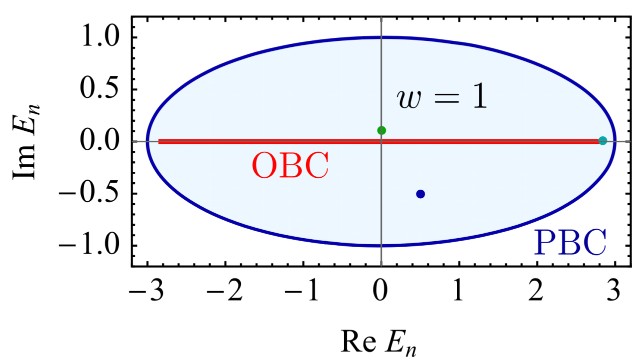
- **Hatano-Nelson model** Hatano & Nelson, PRL **77**, 570 (1996)

asymmetric hopping



$$\hat{H}_{\mathrm{HN}} = \sum_i \left(J_{\mathrm{R}} \hat{c}^\dagger_{i+1} \hat{c}_i + J_{\mathrm{L}} \hat{c}^\dagger_i \hat{c}_{i+1} \right)$$

$$H_{\mathrm{HN}}(k) = J_{\mathrm{R}} e^{\mathrm{i}k} + J_{\mathrm{L}} e^{-\mathrm{i}k}$$



Winding of complex energy!

$$W(E) := \oint \frac{dk}{2\pi \mathrm{i}} \frac{d}{dk} \log\det\left(H(k) - E\right)$$

(ill-defined in Hermitian systems)

Gong, Ashida, **KK** *et al.*, PRX **8**, 031079 (2018)

☆ **Skin effect: bulk-boundary correspondence for point-gap topology**

Emergence of an extensive number of boundary modes!

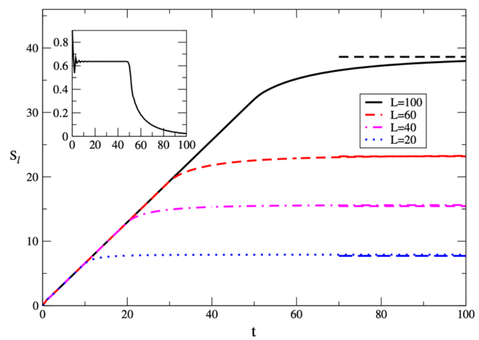
Okuma, **KK**, Shiozaki & Sato, PRL **124**, 086801 (2020); Zhang, Yang & Fang, PRL **125**, 126402 (2020)

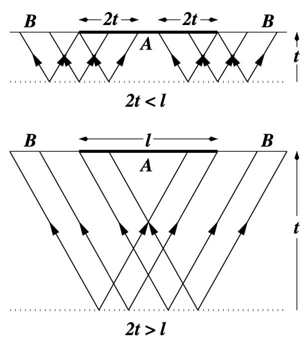
Entanglement dynamics

☆ **Entanglement gives important information on the quantum dynamics.**

closed quantum systems

Calabrese & Cardy, J. Stat. Phys. P04010 (2005)



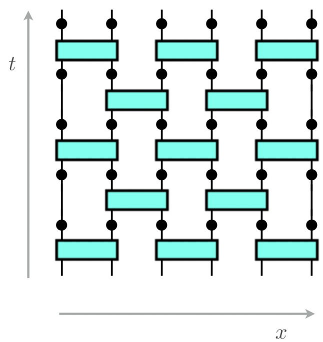


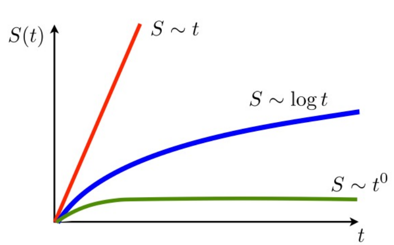
Steady-state entanglement is proportional to the volume of the subsystem (**volume law**)

(Related to thermalization)

Skinner *et al.*, PRX **9**, 031009 (2019);
Li *et al.*, PRB **98**, 205136 (2018)

open quantum systems





Entanglement phase transition as a competition between **unitary dynamics and quantum measurements**

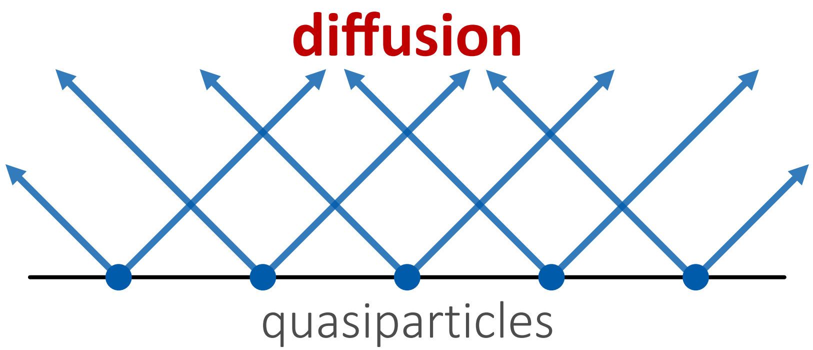
volume law ↔ area law

☆ **In 1+1 D, conformal field theory describes the entanglement dynamics.**

Skin dynamics

closed quantum systems

Calabrese & Cardy, J. Stat. Phys. P04010 (2005)

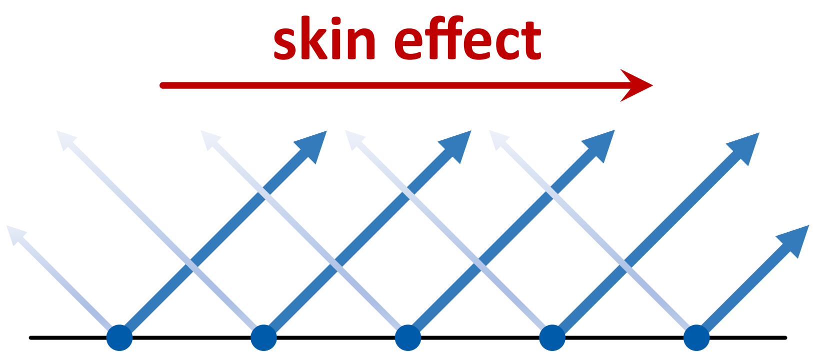


quantum diffusion (thermalization)

→ entanglement propagation

volume law $S \propto l^d$

open quantum systems with the skin effect



skin effect

→ **no diffusion (no thermalization)!**

area law $S \propto l^{d-1}$

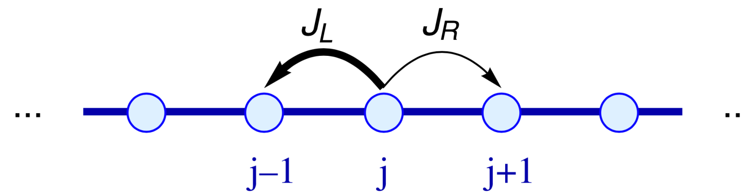
Kawabata, Numasawa & Ryu, PRX **13**, 021007 (2023)

Hatano-Nelson model

We confirm the entanglement suppression for the Hatano-Nelson model

$$\hat{H} = -\frac{1}{2}\sum_{l}\left[(J+\gamma)\,\hat{c}_{l+1}^{\dagger}\hat{c}_{l} + (J-\gamma)\,\hat{c}_{l}^{\dagger}\hat{c}_{l+1}\right]$$

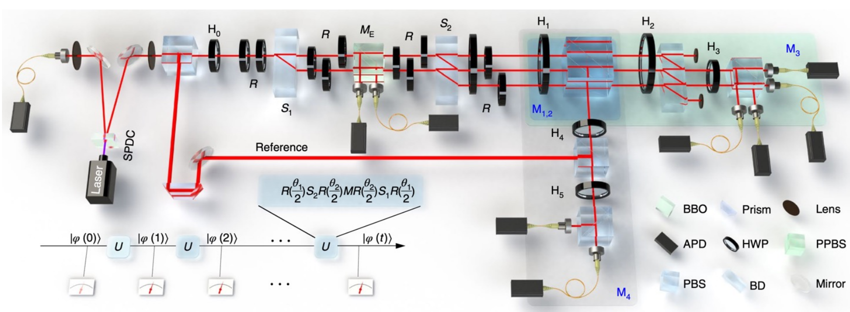
Hatano & Nelson, PRL **77**, 570 (1996)



Gong, Ashida, **Kawabata** *et al.*, PRX **8**, 031079 (2018)

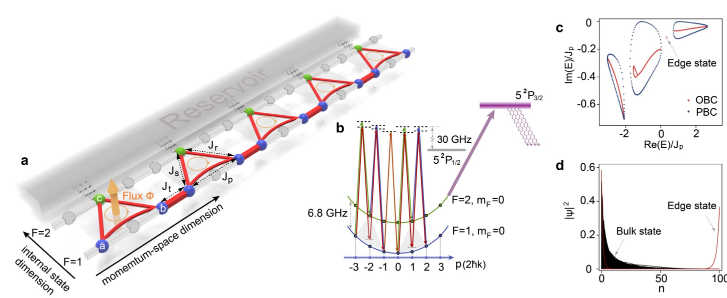
Nonunitary quantum dynamics: $|\psi(t)\rangle = \dfrac{e^{-\mathrm{i}\hat{H}t}|\psi_0\rangle}{\| e^{-\mathrm{i}\hat{H}t}|\psi_0\rangle \|}$, $\quad |\psi_0\rangle = \left(\prod_{l=1}^{L/2}\hat{c}_{2l}^{\dagger}\right)|\mathrm{vac}\rangle$

- **Quantum walk (single photons)**



Xiao *et al.*, Nat. Phys. **16**, 761 (2020)

- **Ultracold atoms**

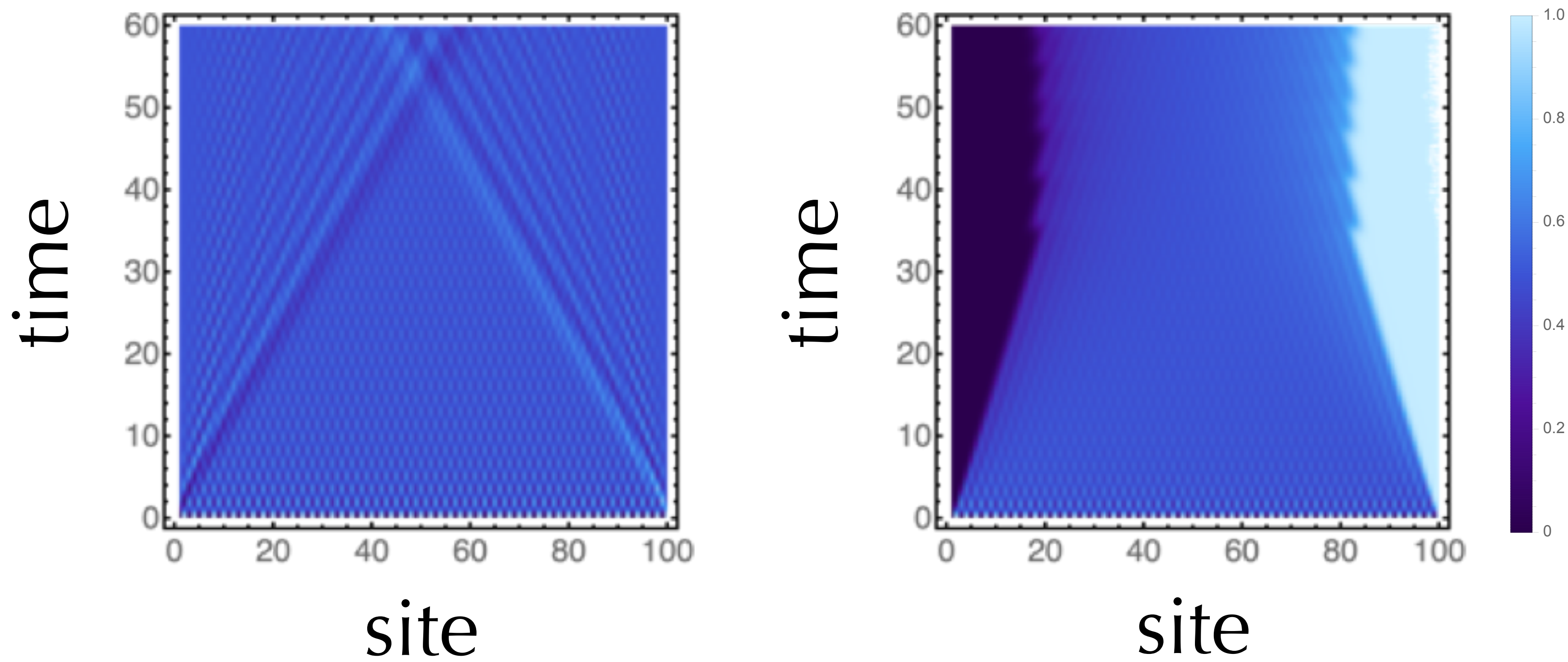


Liang *et al.*, PRL **129**, 070401 (2022)

Skin effect

$(\gamma = 0)$ **Hermitian case**

$(\gamma = 0.8)$ **Non-Hermitian case**

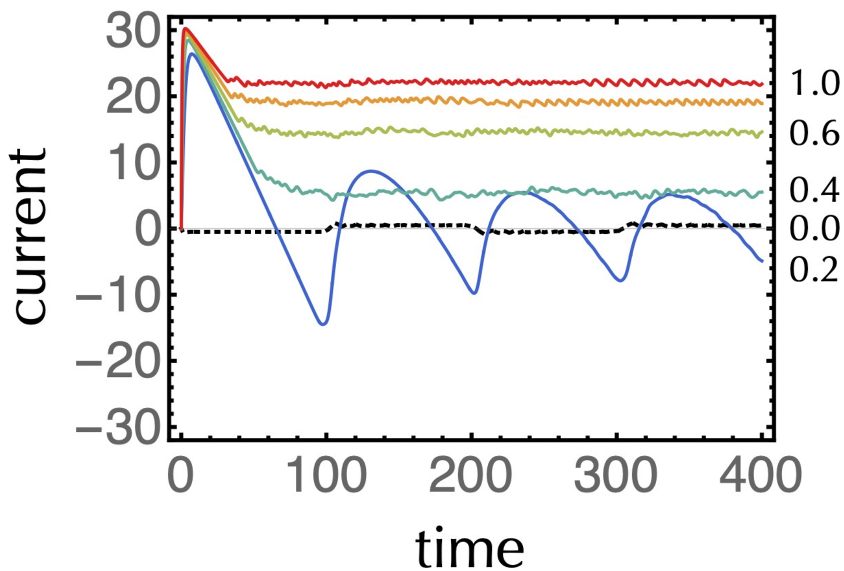


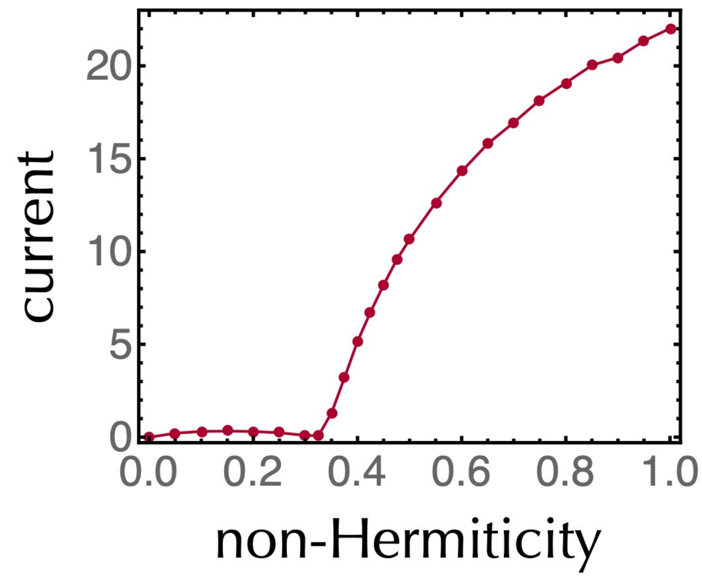
Clear signature of the skin effect!

Plots of the evolutions of the local particle numbers $\langle \psi(t) | \hat{n}_l | \psi(t) \rangle$

Skin current

charge current: $\hat{I} = (\mathrm{i}J/2)\sum_{l=1}^{L-1}\left(\hat{c}_l^\dagger \hat{c}_{l+1} - \hat{c}_{l+1}^\dagger \hat{c}_l\right)$





Nonequilibrium steady state with a nonzero current $I \neq 0$

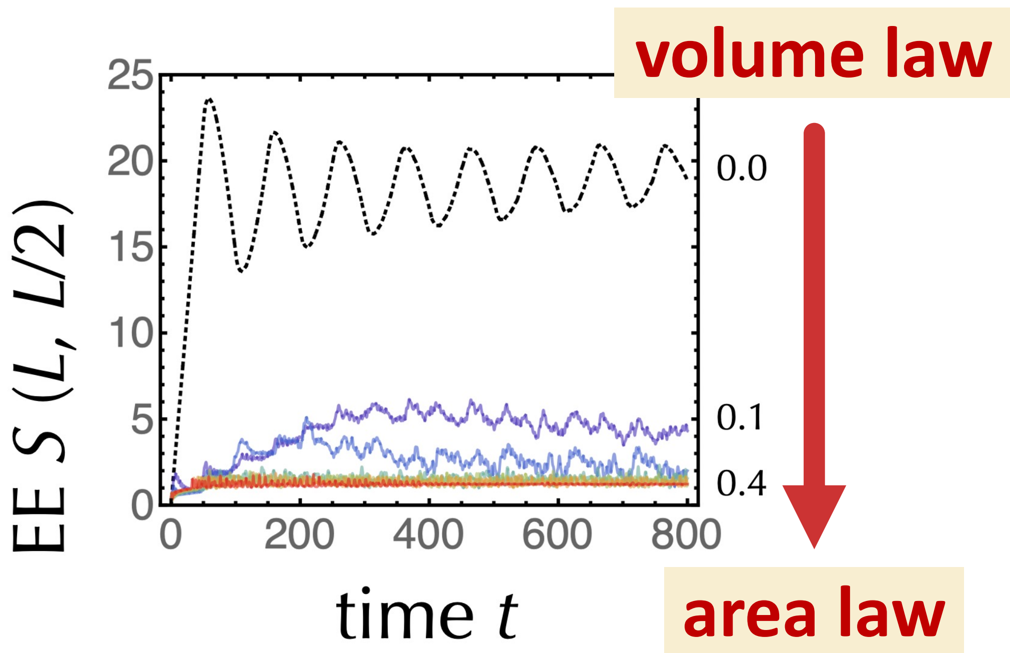
↔ **Thermal equilibrium state:** $I = 0$

cf. Bloch theorem

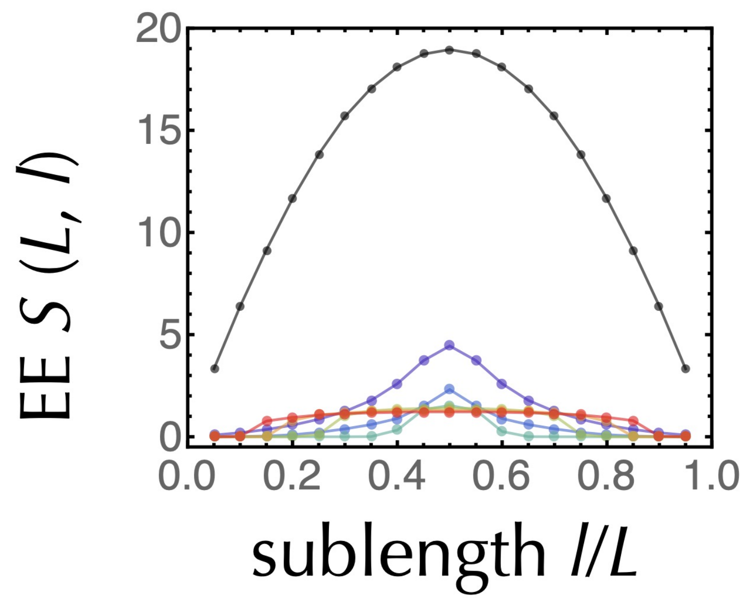
Watanabe, J. Stat. Phys. **177**, 717 (2019)

Entanglement suppression

The skin effect greatly suppresses the entanglement growth!



$(J = 1.0, L = 100)$



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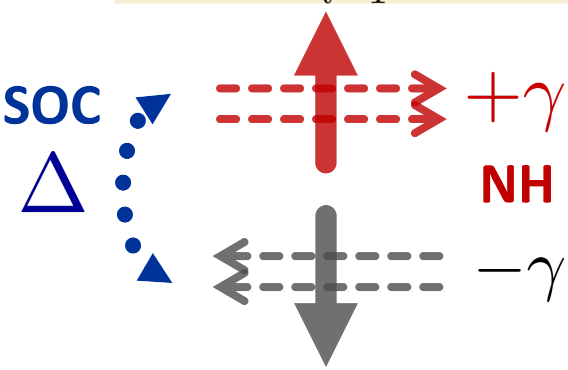
Symplectic Hatano-Nelson model

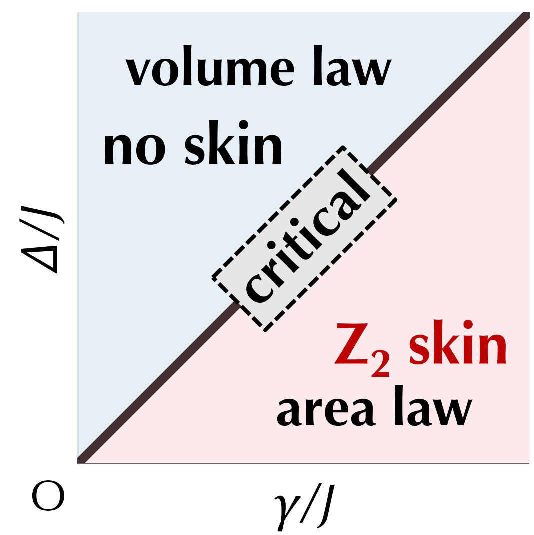
In the Hatano-Nelson model, infinitesimal non-Hermiticity leads to the skin effect.
Can we have a phase transition due to the skin effect?

Symplectic (helical) generalization of the Hatano-Nelson model:

$$\hat{H} = -\frac{1}{2} \sum_{l=1}^{L} \left[\hat{c}_{l+1}^{\dagger} \left(J + \gamma \sigma_z - \mathrm{i} \Delta \sigma_x \right) \hat{c}_l + \hat{c}_l^{\dagger} \left(J - \gamma \sigma_z + \mathrm{i} \Delta \sigma_x \right) \hat{c}_{l+1} \right]$$



Competition between non-Hermiticity and spin-orbit coupling!

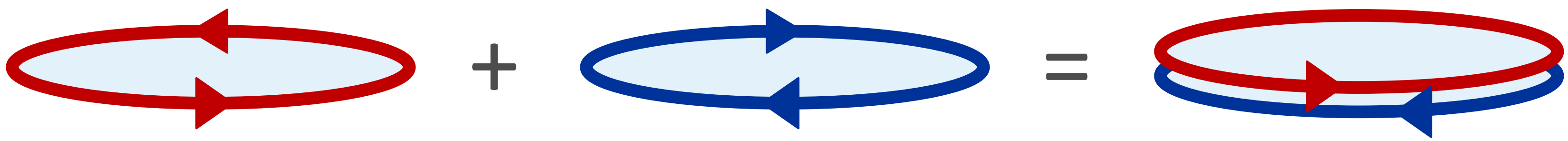


Characterized by a Z_2 topological invariant

Okuma, **Kawabata**, Shiozaki & Sato, PRL **124**, 086801 (2020)
Kawabata, Okuma & Sato, PRB **101**, 195147 (2020)
Kawabata & Ryu, PRL **126**, 166801 (2021)

Z_2 skin effect protected by TRS† (1)

- **Z_2 topological phase in Hermitian systems**

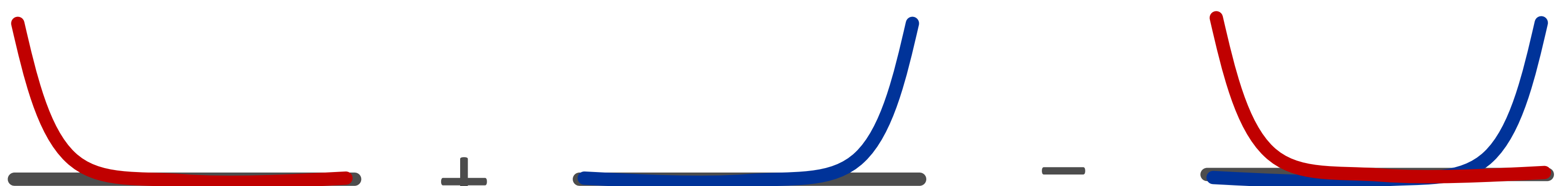


Haldane model
Z invariant (Chern #)
chiral edge states

time-reversed partner

Kane-Mele model
Z_2 invariant with TRS
helical edge states

- **Z_2 skin effect in non-Hermitian systems**



localized at left (+W)

localized at right (-W)

localized at both!

Hatano-Nelson model
Z invariant (point gap)

time-reversed partner
(reciprocal)

Reciprocal skin effect with the Z_2 invariant protected by time-reversal symmetry†

Z_2 skin effect protected by TRS† (2)

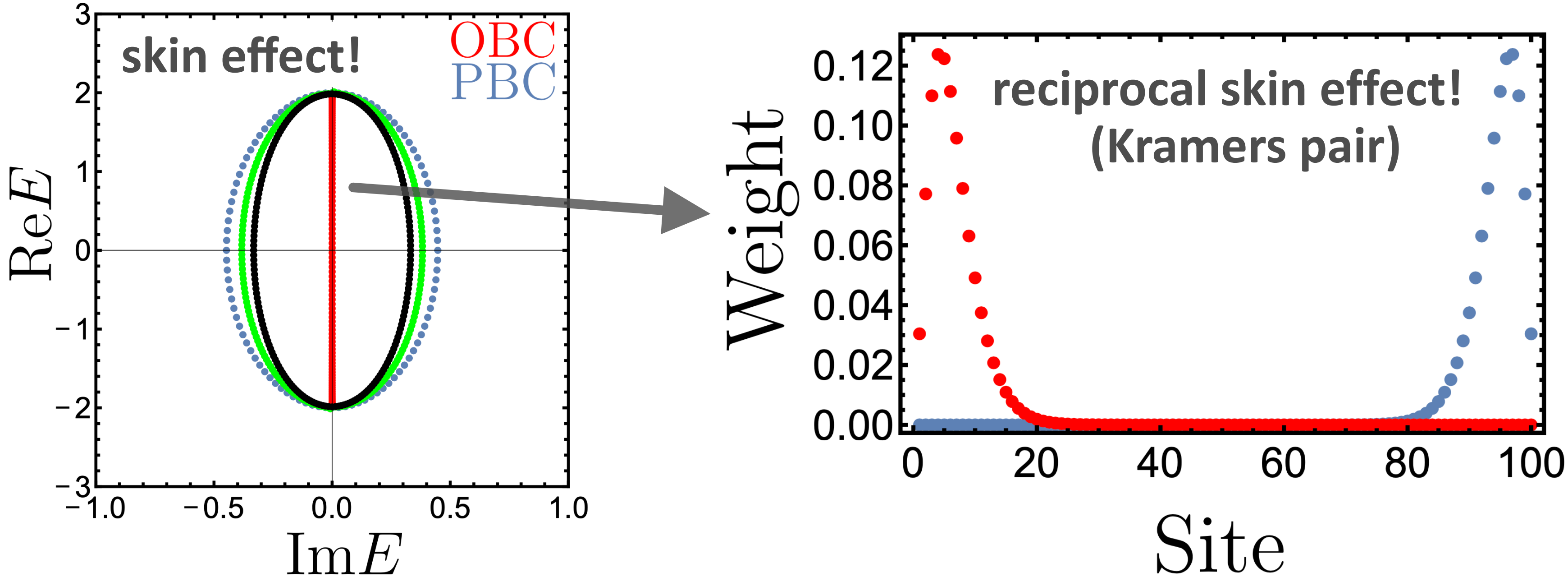
$$H(k) = \begin{pmatrix} H_{\mathrm{HN}}(k) & 2\Delta \sin k \\ 2\Delta \sin k & H_{\mathrm{HN}}^T(-k) \end{pmatrix} = 2t\cos k + 2\Delta (\sin k)\,\sigma_x + 2\mathrm{i}g(\sin k)\,\sigma_z,$$

(symmetry-preserving perturbation)

Okuma, **KK**, Shiozaki & Sato, PRL **124**, 086801 (2020)

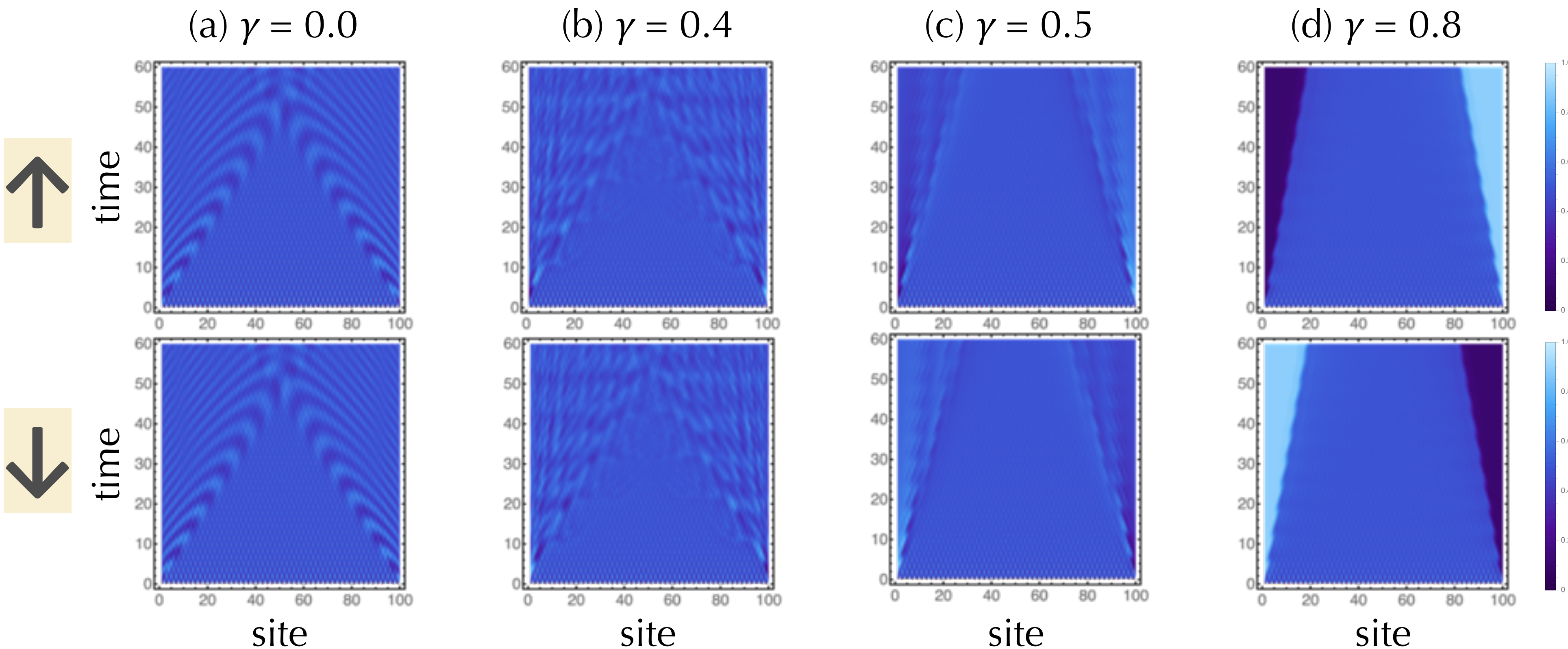
$$\mathrm{TRS}^\dagger:\ (\mathrm{i}\sigma_y)\,H^T(k)\,(\mathrm{i}\sigma_y)^{-1} = H(-k),\quad (\mathrm{i}\sigma_y)(\mathrm{i}\sigma_y)^* = -1$$

→ Z_2 topological, Kramers degeneracy



Reciprocal skin effect

$(J = 1.0, \Delta = 0.5)$



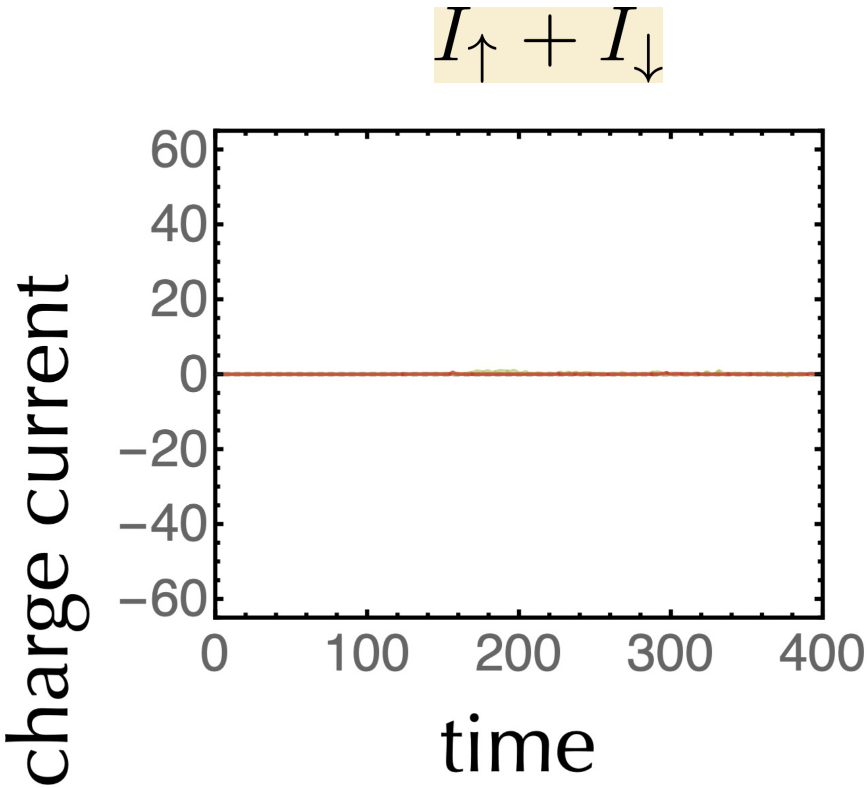
Reciprocal skin effect!

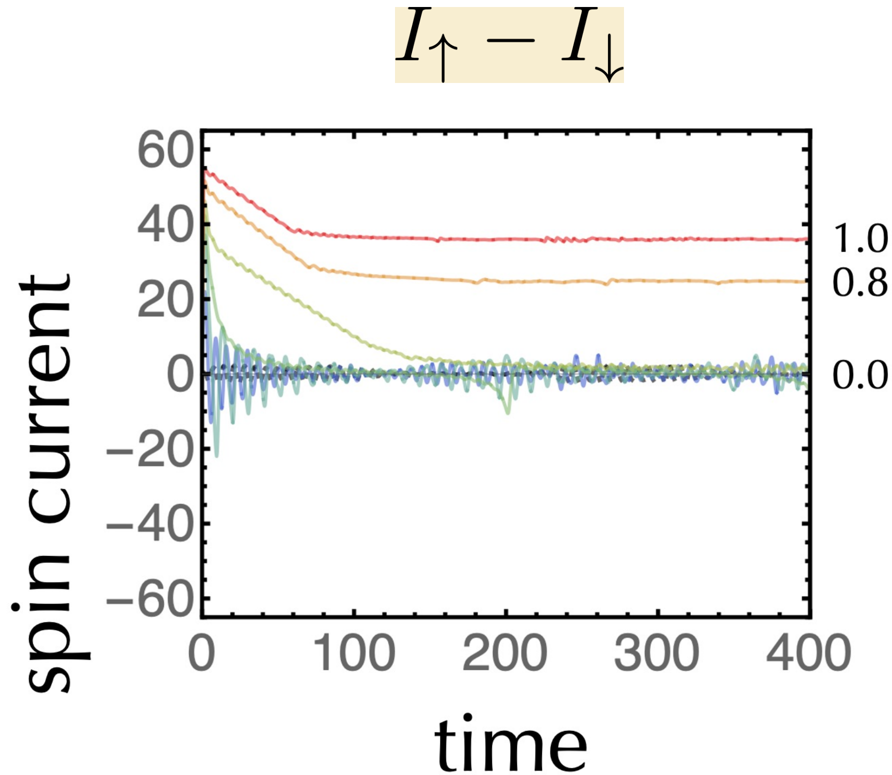
Up spins are localized toward the right edge, and down spins are localized toward the left edge.

Okuma, **Kawabata**, Shiozaki & Sato, PRL **124**, 086801 (2020); **Kawabata**, Okuma & Sato, PRB **101**, 195147 (2020)

Spin current

$(J = 1.0, \Delta = 0.5)$

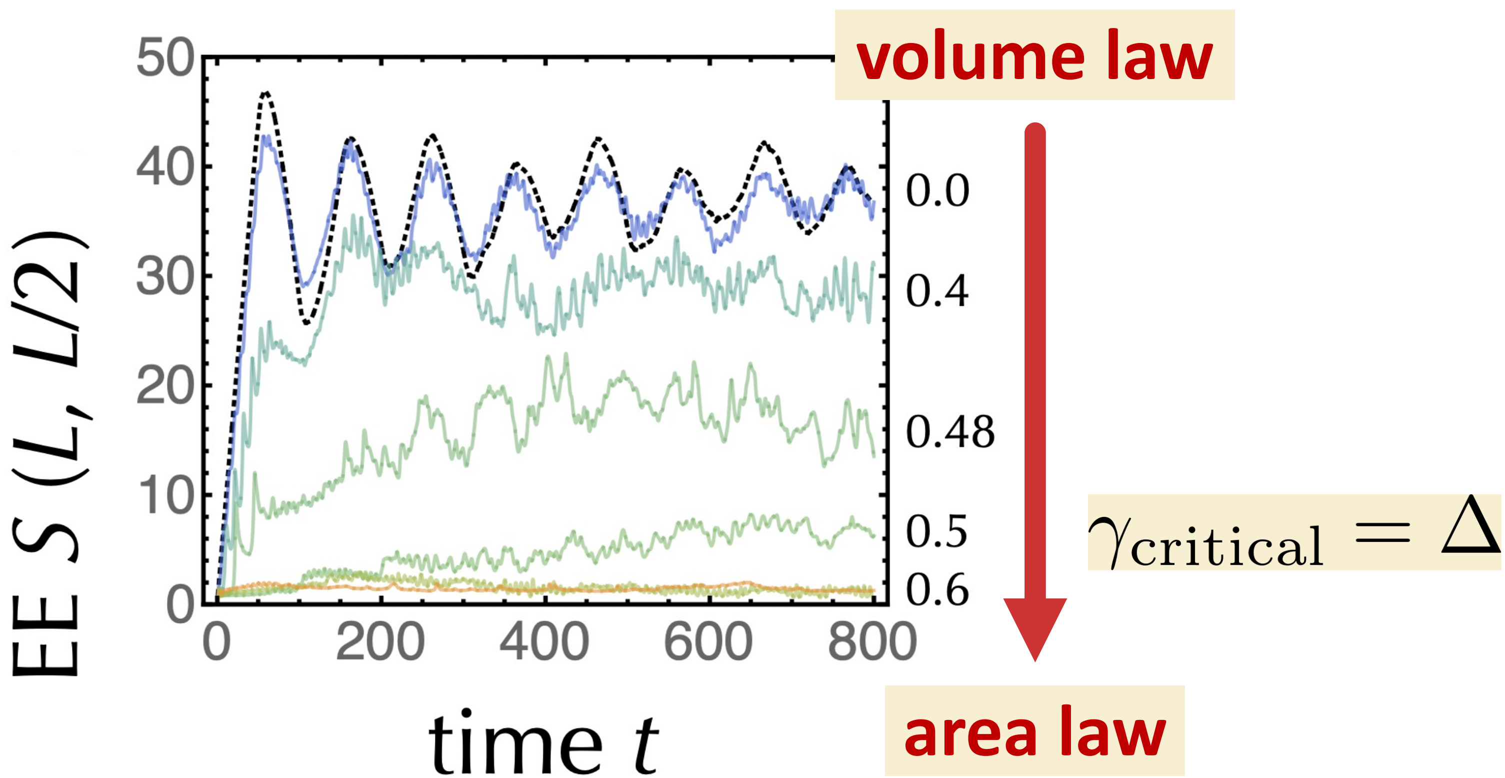




The nonequilibrium steady state is characterized by a nonzero spin current!

Entanglement phase transition

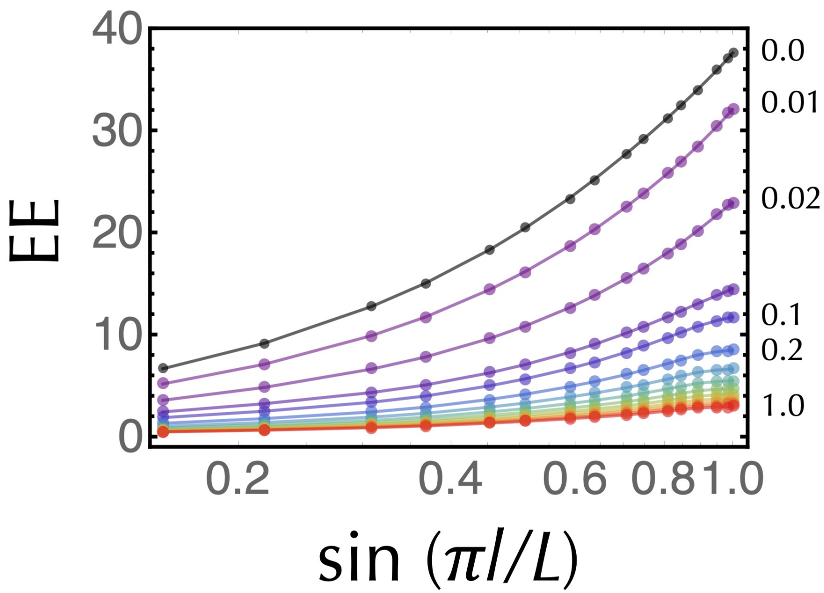
The skin effect induces an entanglement phase transition!

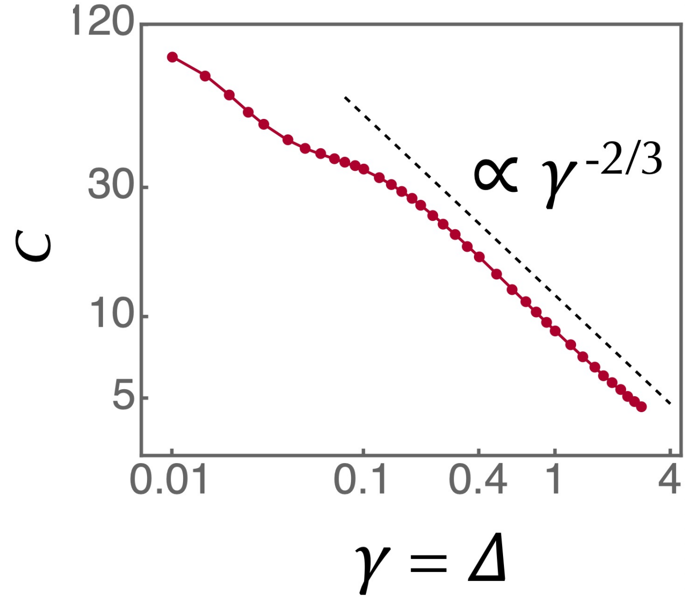


$(J = 1.0, \Delta = 0.5)$

Nonequilibrium quantum criticality

The steady-state entanglement entropy at the critical point $\gamma = \Delta$





The entanglement entropy is well fitted with the CFT description

$$S_c = \frac{c}{6} \log\left(\sin\frac{\pi l}{L}\right) + S_0$$

Calabrese & Cardy, J. Stat. Phys. P06002 (2004)

However, the central charge is parameter dependent $c \propto \gamma^{-2/3}$

→ **nonunitary (irrational) CFT**

Critical skin effect

What is the origin of the nonequilibrium quantum criticality?

→ **Scale invariance of the skin modes!**

☆ **Around the critical point, the localization length diverges.**

$$\xi = \frac{J}{\sqrt{\gamma^2 - \Delta^2}} \propto (\gamma - \gamma_c)^{-1/2}$$

☆ **At the critical point, the skin modes decay with the power law.**

$$\phi_l \simeq \frac{\gamma l}{J} \phi_0$$

The power-law decay arises from an exceptional point!

Single-particle Schrödinger equation $-\frac{J}{2}\begin{pmatrix} 1 & \gamma/J \\ 0 & 1 \end{pmatrix} \phi_{l-1} - \frac{J}{2}\begin{pmatrix} 1 & -\gamma/J \\ 0 & 1 \end{pmatrix} \phi_{l+1} = E\phi_l$

$$\longrightarrow \phi_l \sim \underbrace{\begin{pmatrix} 1 & \gamma/J \\ 0 & 1 \end{pmatrix}^l}_{} \phi_0 \simeq \frac{\gamma l}{J} \phi_0$$

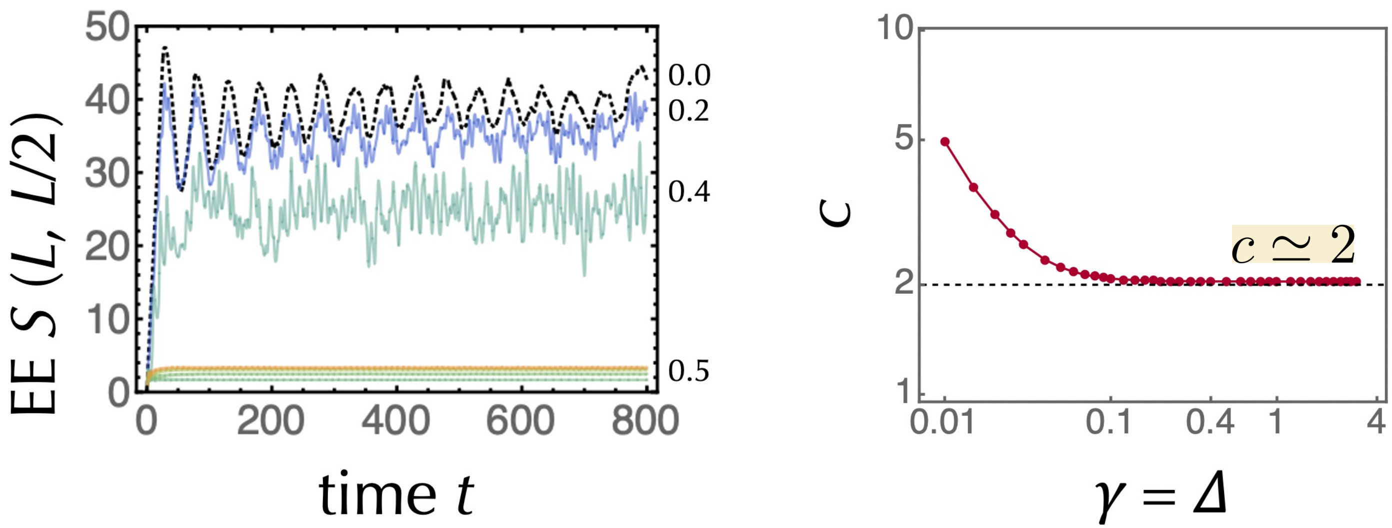
Jordan matrix (nondiagonalizable)

☆ **New type of critical phenomena unique to open quantum systems.**

Periodic boundary conditions

The skin effect is crucial for the universality class.

Entanglement dynamics under the PBC:



The CFT scaling is well, but the central charge is a constant!

Mechanisms of the nonequilibrium quantum criticality:

OBC: critical skin effect (scale-invariant skin modes due to an EP)

PBC: no skin effect occurs; real-complex spectral transition

The scale invariance at the critical point looks similar to the critical phenomena at thermal equilibrium.

$$\xi \propto |T - T_{\mathrm{c}}|^{-\nu}, \quad \langle S(r) S(0) \rangle \propto r^{-(d-2+\eta)}$$

However, our phase transitions provide a new nonequilibrium quantum criticality unique to open quantum systems.

(mechanism: an exceptional point and concomitant scale-invariant skin modes)

New universality class of nonequilibrium quantum phase transition in open quantum systems.

Outline

1. Introduction

2. Entanglement suppression induced by the non-Hermitian skin effect

3. Entanglement phase transition induced by the non-Hermitian skin effect

4. Purification induced by the non-Hermitian skin effect in Lindbladians

Liouvillian skin effect

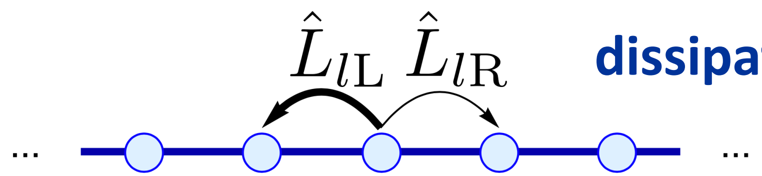
☆ **The skin effect occurs also in the quantum master equation.**

Song, Yao & Wang, PRL **123**, 170401 (2019)

$$\frac{d}{dt}\hat{\rho} = \sum_{l=1}^{L} \sum_{n=\mathrm{R,L}} \left(\hat{L}_{ln}\hat{\rho}\hat{L}_{ln}^{\dagger} - \frac{1}{2}\{\hat{L}_{ln}^{\dagger}\hat{L}_{ln}, \hat{\rho}\} \right)$$

Znidaric, PRE **92**, 042143 (2015);
Haga *et al.*, PRL **127**, 070402 (2021)

$$\hat{L}_{l\mathrm{R}} = \sqrt{\frac{J+\gamma}{2}}\,\hat{c}_{l+1}^{\dagger}\hat{c}_{l}, \quad \hat{L}_{l\mathrm{L}} = \sqrt{\frac{J-\gamma}{2}}\,\hat{c}_{l}^{\dagger}\hat{c}_{l+1}$$



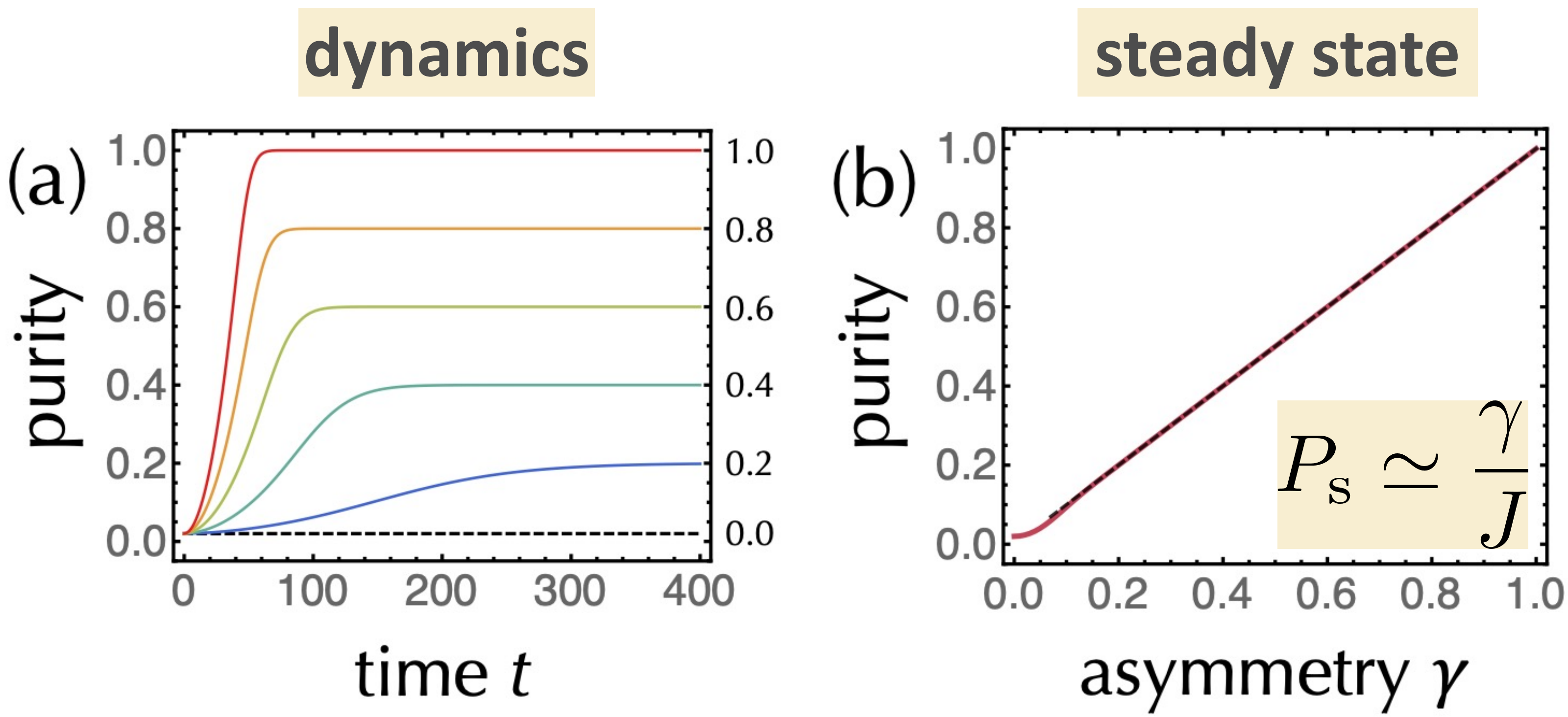
dissipative hopping

Initial state: completely mixed state $\hat{\rho}_0 = 1/L$
(single-particle subspace)

Open quantum dynamics subject to the skin effect?

Purification

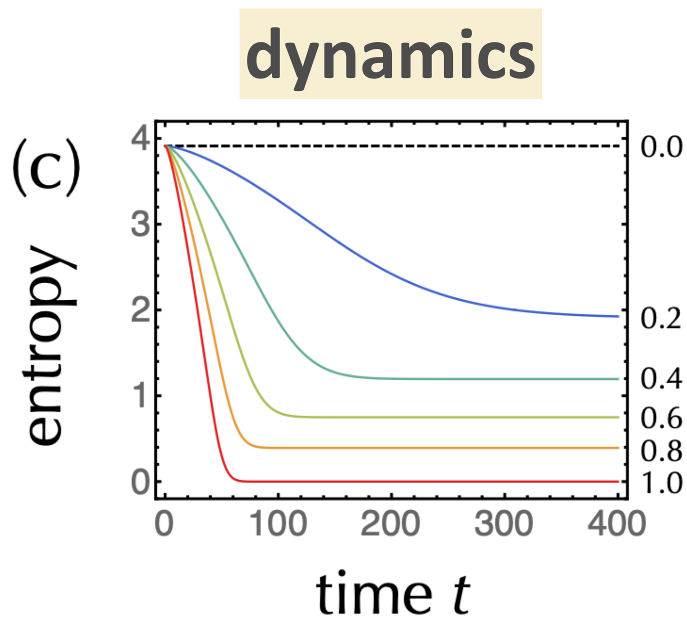
The skin effect induces purification!

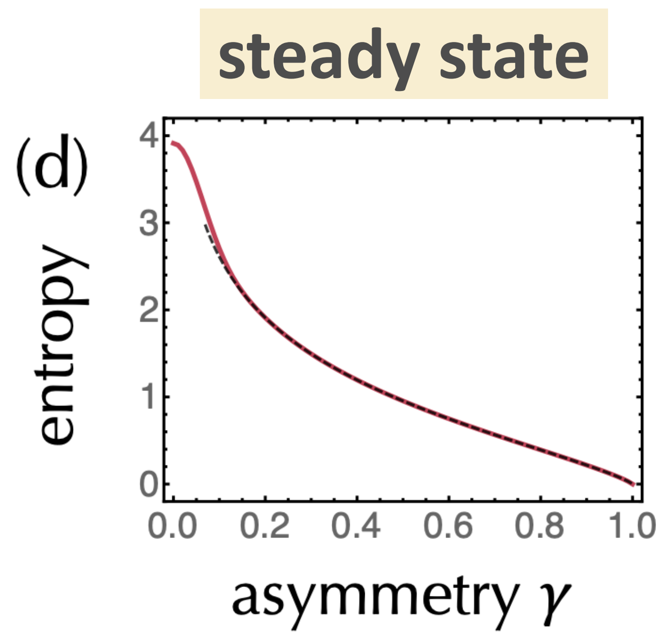


$$\hat{\rho}_s \propto \sum_{l=1}^{L} r^l \left| l \right\rangle \left\langle l \right| \quad \left(r := \frac{J+\gamma}{J-\gamma} \right)$$

Suppression of von Neumann entropy

The skin effect also suppresses von Neumann entropy!





Purification can arise also in the conditional dynamics under quantum measurements.

Gullans & Huse, PRX **10**, 041020 (2020)

⟷ **The skin effect induces purification even in the master equation!**
(averaged over multiple quantum trajectories)

Summary

Phys. Rev. X 13, 021007 (2023)

- The skin effect prohibits the entanglement growth and thermalization, leading to the area law of the entanglement entropy.
- The skin effect triggers a new type of entanglement phase transition that is characterized by an anomalous nonunitary CFT.
- The skin effect leads to the purification also in the master equation.