Entanglement Phase Transition Induced by the Non-Hermitian Skin Effect

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arXiv:2206.05384

Non-Hermitian physics

Conventionally, physicists focused on Hermitian systems at equilibrium.

→ **Richer properties appear in non-Hermitian systems!**

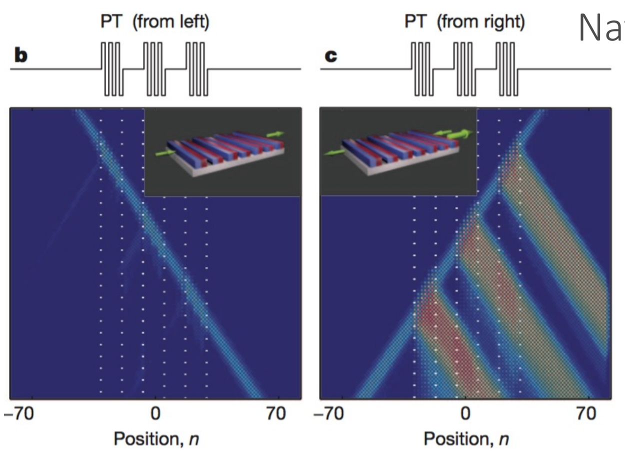
☆ Non-Hermiticity arises from **dissipation**, i.e., exchanges of energy or particles with an environment.

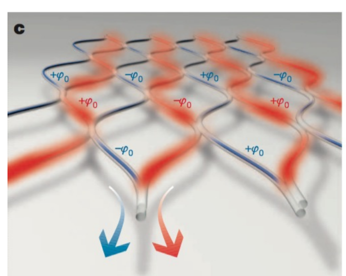
El-Ganainy *et al.*, Nat. Phys. **14**, 11 (2018)

- **Photonic lattices with gain/loss**

Unidirectional light transport

Regensburger *et al.*, Nature **488**, 167 (2012)

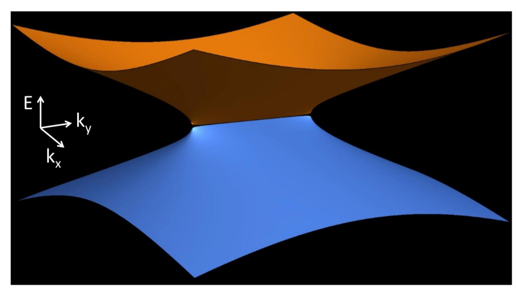


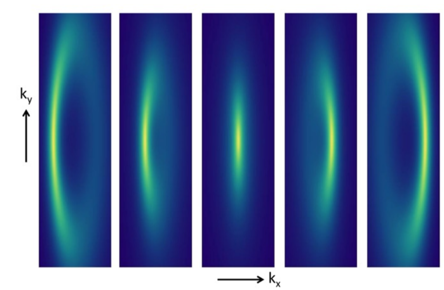


- **Finite-lifetime quasiparticles**

Bulk Fermi arc due to non-Hermitian self-energy

Kozii & Fu, arXiv: 1708.05841



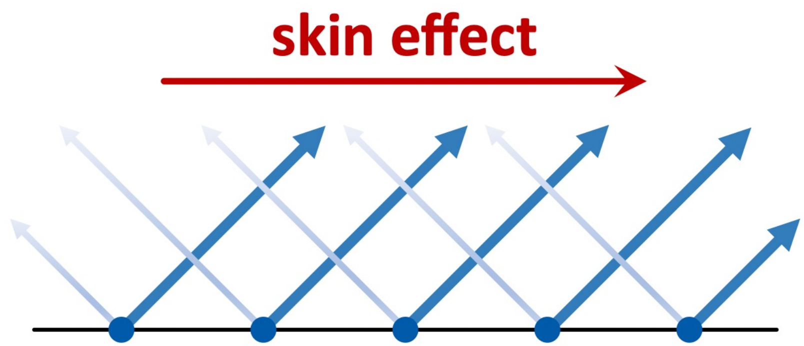


Non-Hermitian skin effect

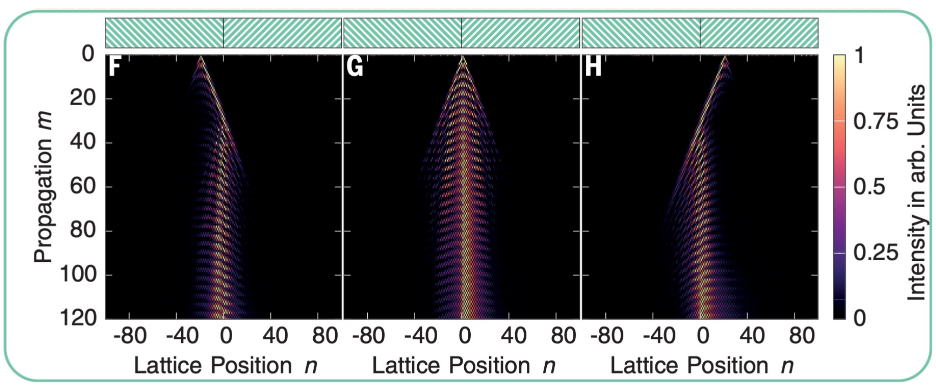
☆ Non-Hermitian skin effect

Lee, PRL **116**, 133903 (2016); Yao & Wang, PRL **121**, 086803 (2018)

Localization of an extensive number of eigenmodes due to non-Hermitian topology

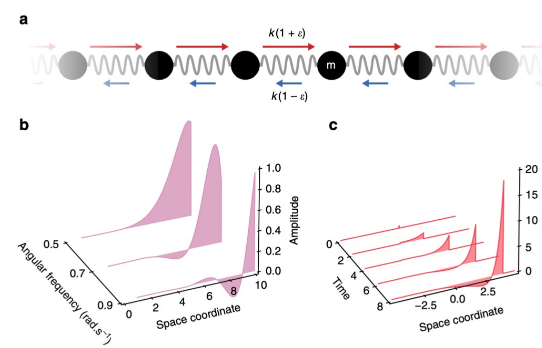


- **Photonic lattice**



Weidemann *et al.*, Science **368**, 311 (2020)

- **Mechanical metamaterials**



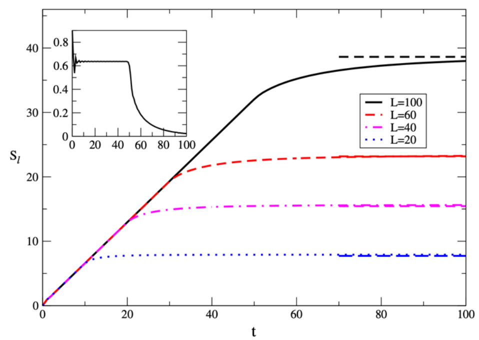
Brandenbourger *et al.*, Nat. Commun. **10**, 4608 (2019)

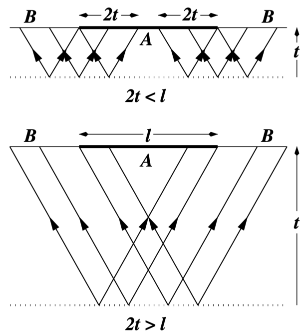
Entanglement dynamics

☆ **Entanglement gives important information on the quantum dynamics.**

closed quantum systems

Calabrese & Cardy, J. Stat. Phys. P04010 (2005)



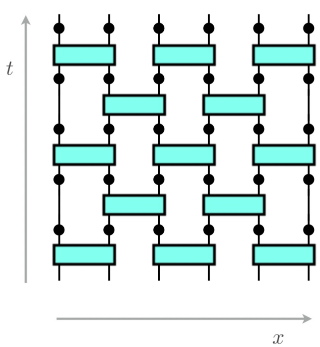


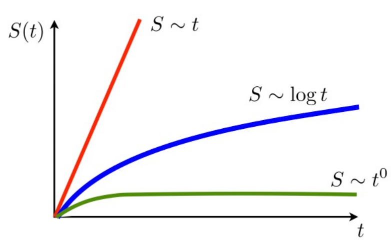
Steady-state entanglement is proportional to the volume of the subsystem (**volume law**)

(Related to thermalization)

Chan *et al.*, PRB **99**, 224307 (2019);
Skinner *et al.*, PRX **9**, 031009 (2019);
Li *et al.*, PRB **98**, 205136 (2018)

open quantum systems





Entanglement phase transition as a competition between **unitary dynamics and quantum measurements**

volume law ↔ area law

☆ **In 1+1 D, conformal field theory describes the entanglement dynamics.**

Motivation and Results

The nature of entanglement in open quantum systems has remained largely elusive.

We show that the **non-Hermitian skin effect** plays an important role in the **entanglement dynamics of open quantum systems**.

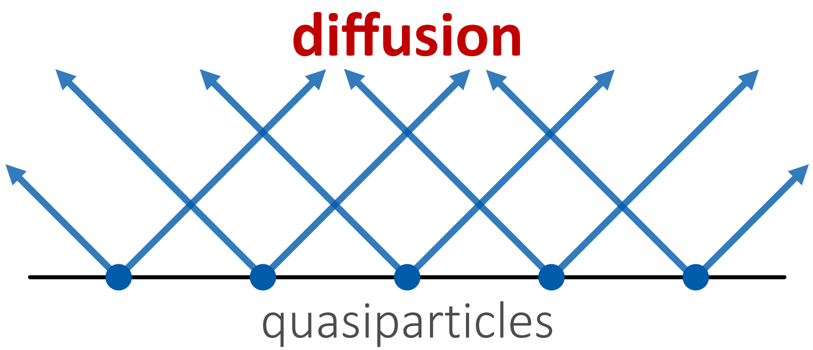
(1) The skin effect **prohibits the entanglement propagation and thermalization**, leading to the area law for the steady state.

(2) The skin effect even triggers a **new type of entanglement phase transition** characterized by a new nonunitary CFT.

Skin dynamics

closed quantum systems

Calabrese & Cardy, J. Stat. Phys. P04010 (2005)

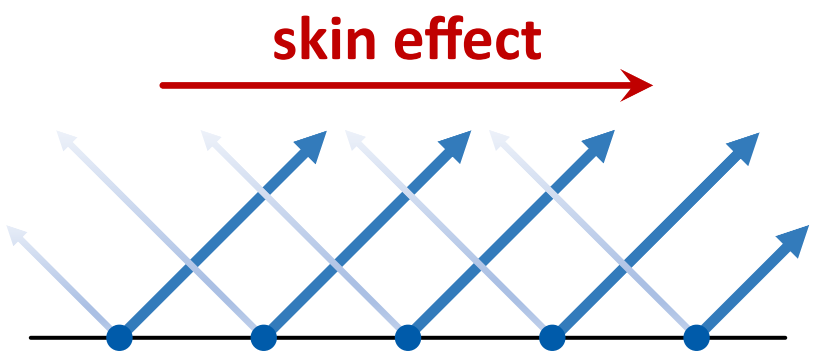


quantum diffusion (thermalization)

→ entanglement propagation

volume law $S \propto l^d$

open quantum systems with the skin effect



skin effect

→ **no diffusion (no thermalization)!**

area law $S \propto l^{d-1}$

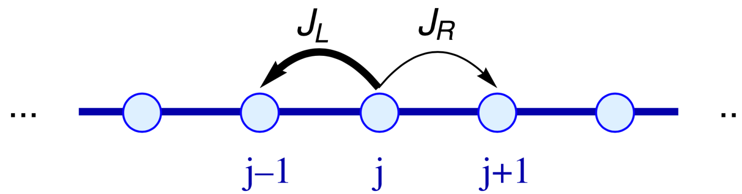
Kawabata, Numasawa & Ryu, arXiv:2206.05384

Hatano-Nelson model

We confirm the entanglement suppression for the Hatano-Nelson model

$$\hat{H} = -\frac{1}{2}\sum_{l}\left[\left(J+\gamma\right)\hat{c}_{l+1}^{\dagger}\hat{c}_{l} + \left(J-\gamma\right)\hat{c}_{l}^{\dagger}\hat{c}_{l+1}\right]$$

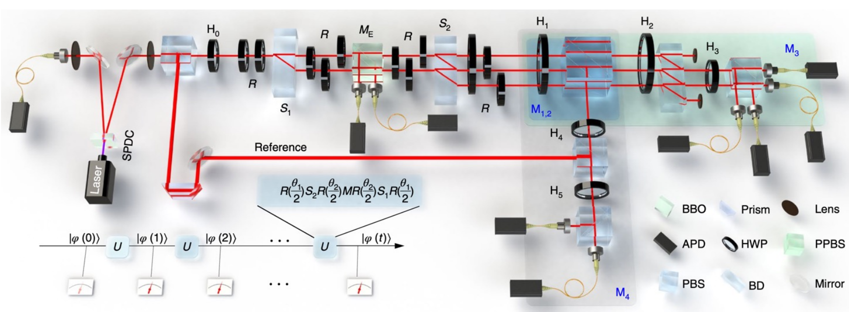
Hatano & Nelson, PRL **77**, 570 (1996)



Gong, Ashida, **Kawabata** *et al.*, PRX **8**, 031079 (2018)

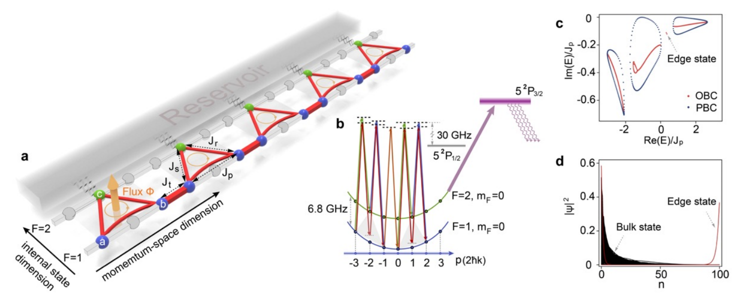
Nonunitary quantum dynamics: $|\psi(t)\rangle = \frac{e^{-\mathrm{i}\hat{H}t}|\psi_0\rangle}{\| e^{-\mathrm{i}\hat{H}t}|\psi_0\rangle \|}$, $|\psi_0\rangle = \left(\prod_{l=1}^{L/2}\hat{c}_{2l}^{\dagger}\right)|\mathrm{vac}\rangle$

- **Quantum walk (single photons)**



Xiao *et al.*, Nat. Phys. **16**, 761 (2020)

- **Ultracold atoms**

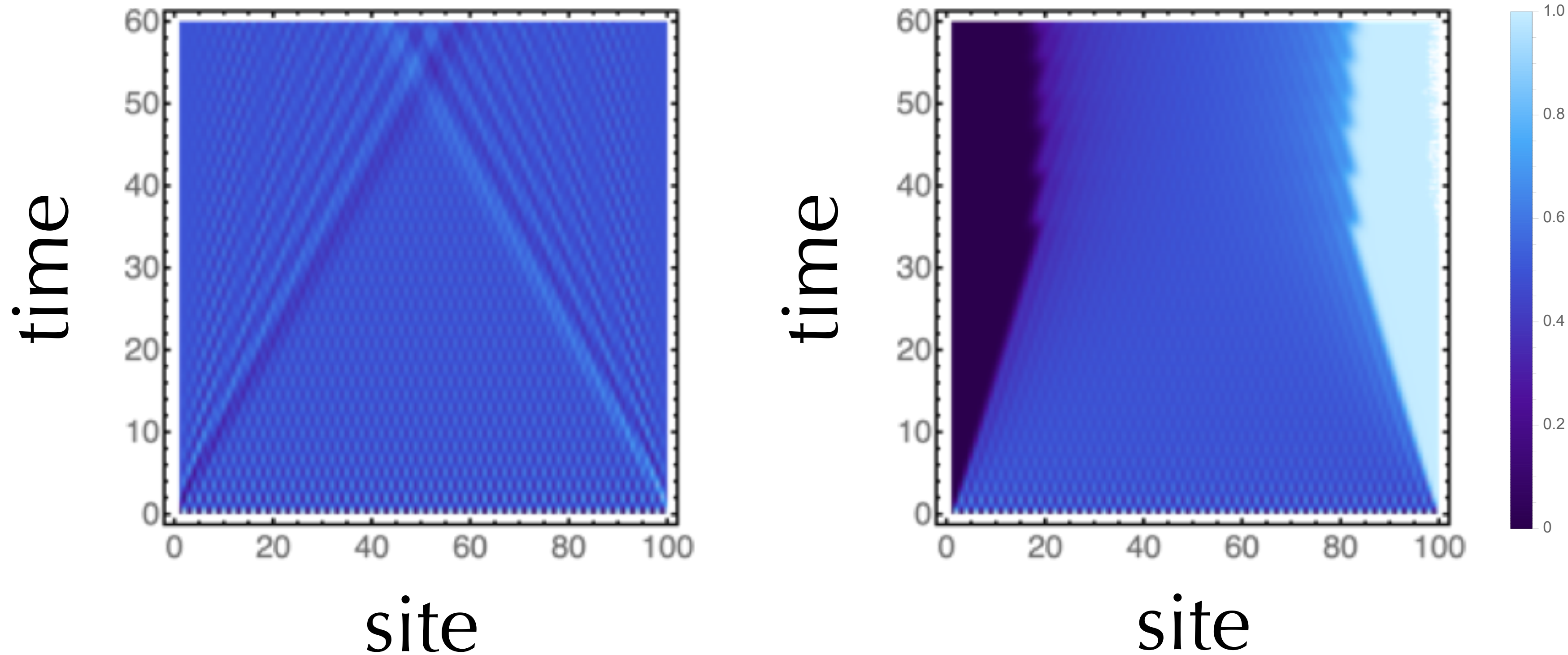


Liang *et al.*, PRL **129**, 070401 (2022)

Skin effect

$(\gamma = 0)$ **Hermitian case**

$(\gamma = 0.8)$ **Non-Hermitian case**

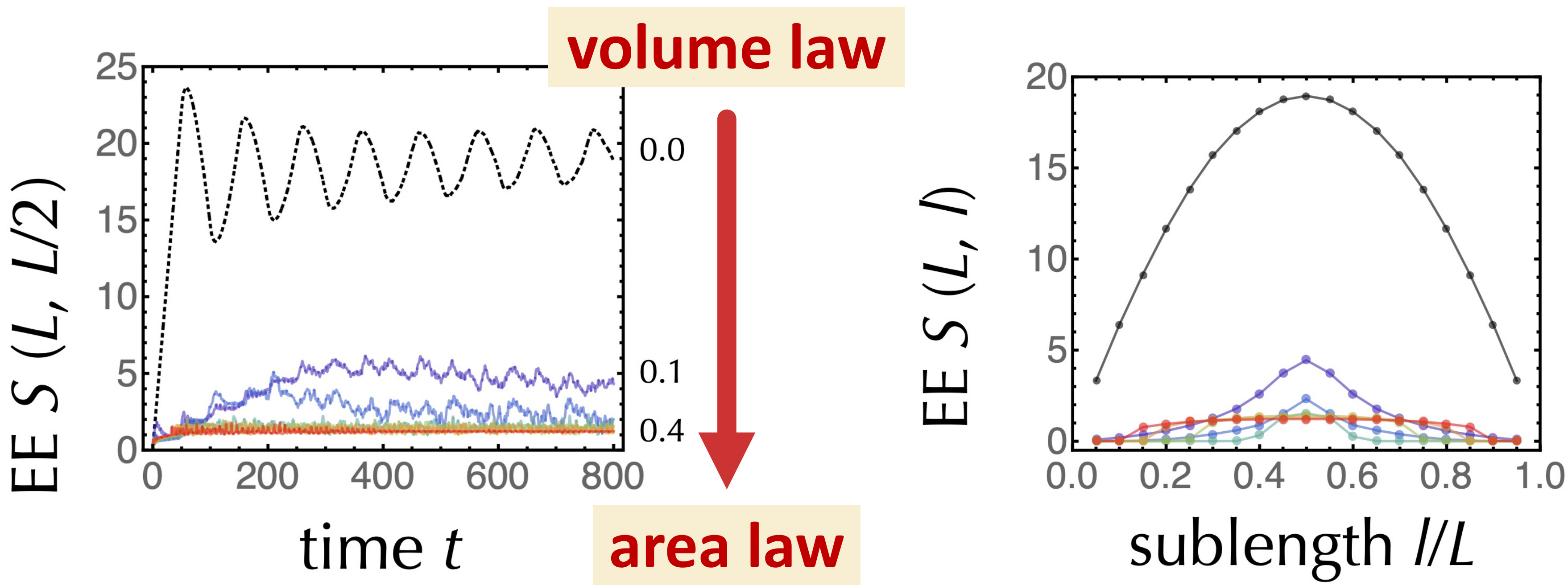


Clear signature of the skin effect!

Plots of the evolutions of the local particle numbers $\langle \psi(t) | \hat{n}_l | \psi(t) \rangle$

Entanglement suppression

The skin effect greatly suppresses the entanglement growth!



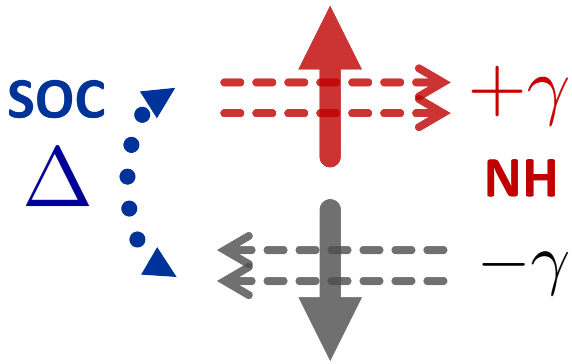
$(J = 1.0, L = 100)$

Symplectic Hatano-Nelson model

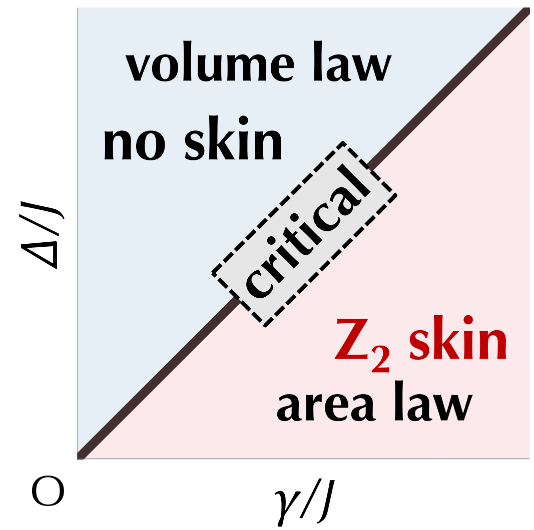
In the Hatano-Nelson model, infinitesimal non-Hermiticity leads to the skin effect.
Can we have a continuous phase transition due to the skin effect?

Symplectic (helical) generalization of the Hatano-Nelson model:

$$\hat{H} = -\frac{1}{2}\sum_{l=1}^{L}\left[\hat{c}_{l+1}^{\dagger}\left(J+\gamma\sigma_z-\mathrm{i}\Delta\sigma_x\right)\hat{c}_l+\hat{c}_l^{\dagger}\left(J-\gamma\sigma_z+\mathrm{i}\Delta\sigma_x\right)\hat{c}_{l+1}\right]$$



Competition between non-Hermiticity and spin-orbit coupling!

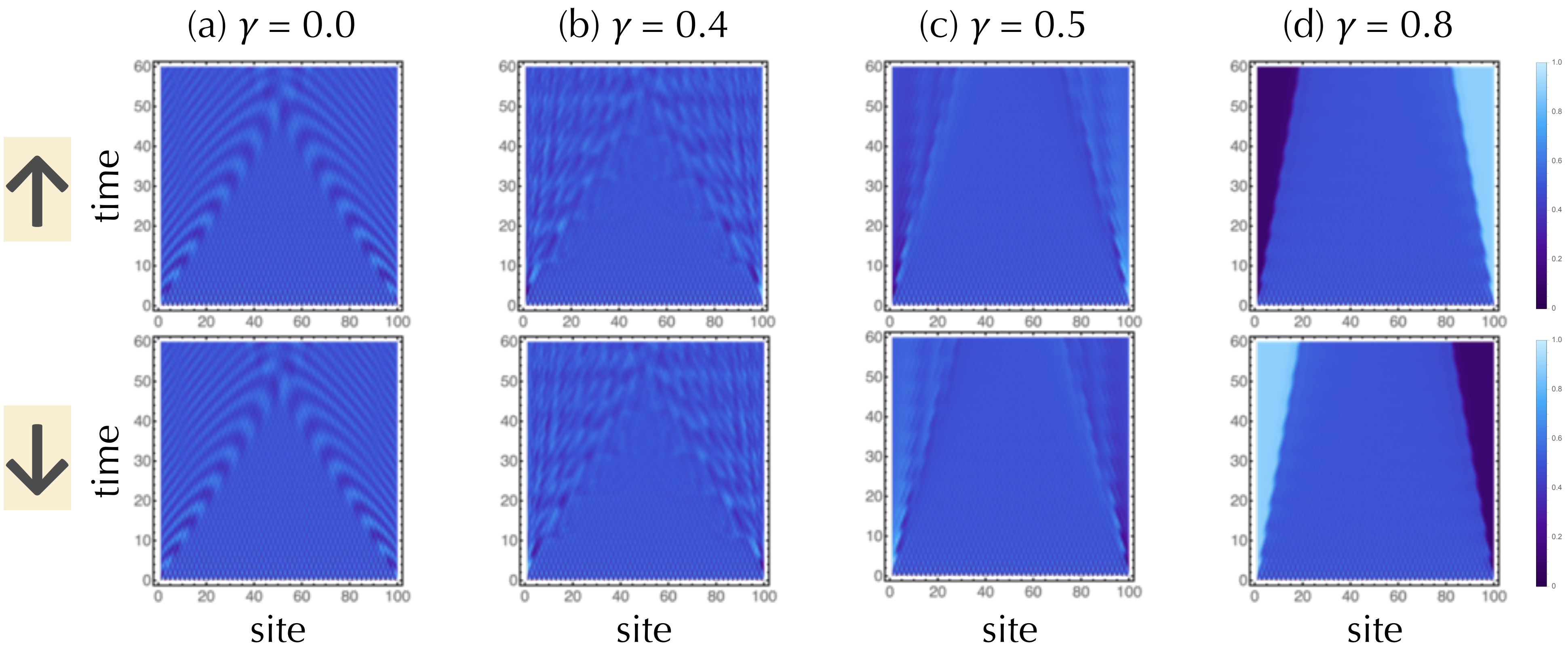


Characterized by a Z_2 topological invariant

Okuma, **Kawabata**, Shiozaki & Sato, PRL **124**, 086801 (2020)
Kawabata, Okuma & Sato, PRB **101**, 195147 (2020)
Kawabata & Ryu, PRL **126**, 166801 (2021)

Reciprocal skin effect

$(J = 1.0, \Delta = 0.5)$



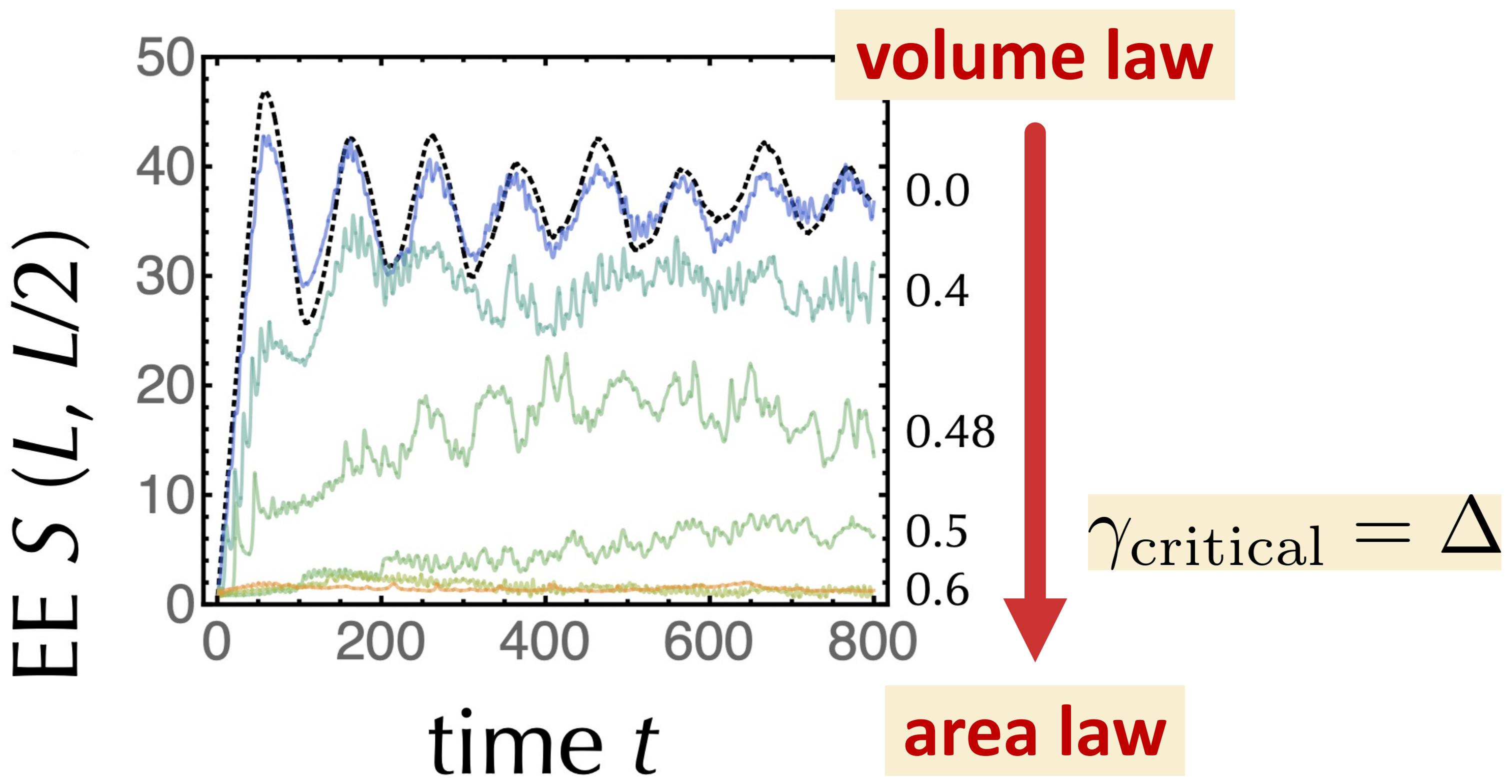
Reciprocal skin effect!

Up spins are localized toward the right edge,
and down spins are localized toward the left edge.

Okuma, **Kawabata**, Shiozaki & Sato, PRL **124**, 086801 (2020); **Kawabata**, Okuma & Sato, PRB **101**, 195147 (2020)

Entanglement phase transition

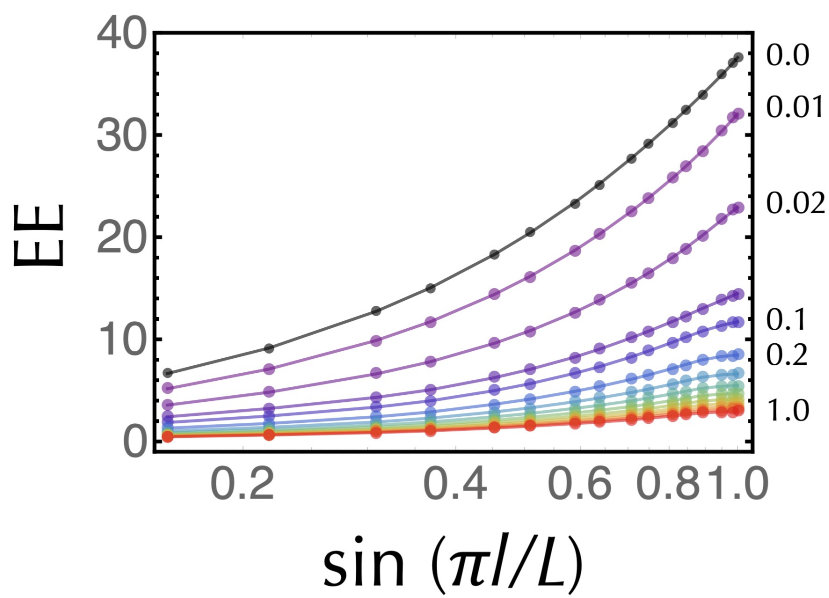
The skin effect induces an entanglement phase transition!

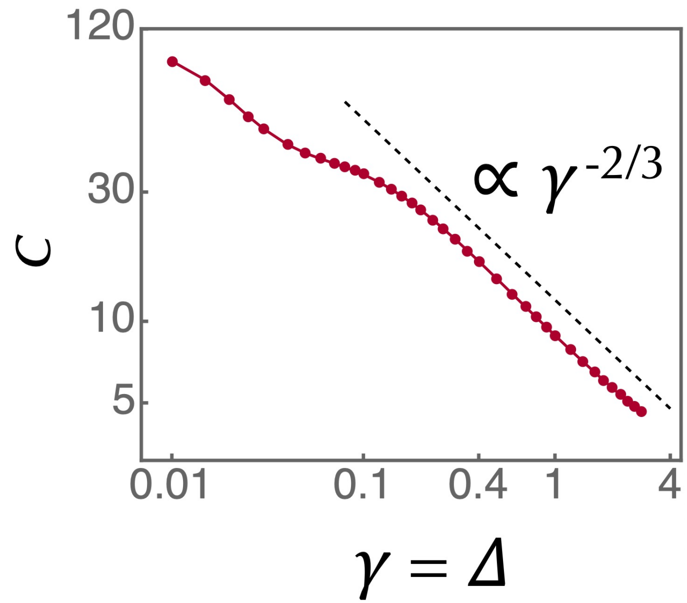


$(J = 1.0, \Delta = 0.5)$

Nonequilibrium quantum criticality

The steady-state entanglement entropy at the critical point $\gamma = \Delta$





The entanglement entropy is well fitted with the CFT description

$$S_c = \frac{c}{6} \log\left(\sin \frac{\pi l}{L}\right) + S_0$$

Calabrese & Cardy, J. Stat. Phys. P06002 (2004)

However, the central charge is parameter dependent $c \propto \gamma^{-2/3}$

→ **nonunitary (irrational) CFT**

Critical skin effect

What is the origin of the nonequilibrium quantum criticality?

→ **Scale invariance of the skin modes!**

☆ **Around the critical point, the localization length diverges.**

$$\xi = \frac{J}{\sqrt{\gamma^2 - \Delta^2}} \propto (\gamma - \gamma_c)^{-1/2}$$

☆ **At the critical point, the skin modes decay with the power law.**

$$\phi_l \simeq \frac{\gamma l}{J} \phi_0$$

The power-law decay arises from an exceptional point!

Single-particle Schrödinger equation $-\frac{J}{2}\begin{pmatrix} 1 & \gamma/J \\ 0 & 1 \end{pmatrix} \phi_{l-1} - \frac{J}{2}\begin{pmatrix} 1 & -\gamma/J \\ 0 & 1 \end{pmatrix} \phi_{l+1} = E\phi_l$

$$\longrightarrow \phi_l \sim \underbrace{\begin{pmatrix} 1 & \gamma/J \\ 0 & 1 \end{pmatrix}^l}_{} \phi_0 \simeq \frac{\gamma l}{J} \phi_0$$

Jordan matrix (nondiagonalizable)

☆ **New type of critical phenomena unique to open quantum systems.**

Summary

arXiv:2206.05384

- The skin effect prohibits the entanglement growth and thermalization, leading to the area law of the entanglement entropy.

- The skin effect triggers a new type of entanglement phase transition that is characterized by an anomalous nonunitary CFT.