東京大学
THE UNIVERSITY OF TOKYO

高次非エルミート表皮効果

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2020 年 9 月 11 日

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arXiv: 2008.07237

Non-Hermitian skin effect

☆ Bulk-boundary correspondence

- Zero modes in the Su-Schrieffer-Heeger model
- Corner zero modes in the Benalcazar-Bernevig-Hughes model

(higher-order topological insulator)

Benalcazar, Bernevig & Hughes, Science **357**, 61 (2017)

☆ Non-Hermitian skin effect

New type of BBC **unique to non-Hermitian systems**.

An **extensive** number of boundary (skin) modes.

Lee, PRL **116**, 133903 (2016)

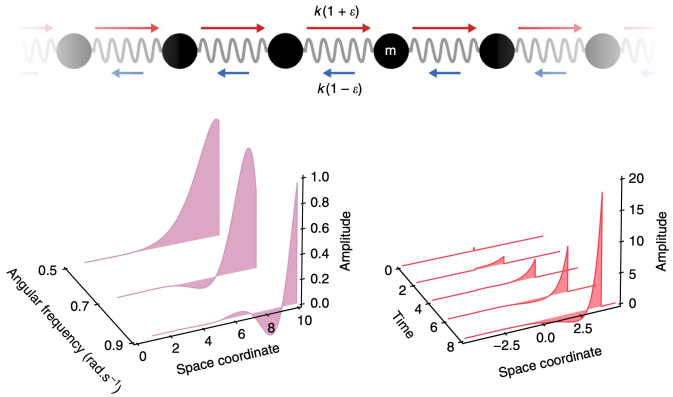
Yao & Wang, PRL **121**, 086803 (2018)

Kunst *et al.*, PRL **121**, 026808 (2018)

Model	Number	Topology
Zero modes in SSH	O (1) from O (L)	Hermitian (line gap)
Corner zero modes in BBH	O (1) from O (L^2)	Hermitian (line gap)
Non-Hermitian skin effect	**O (L) from O (L)**	**Non-Hermitian (point gap)**

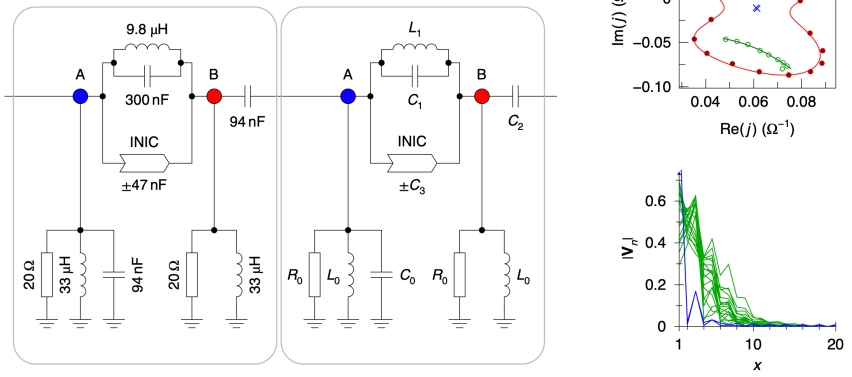
Skin effect in experiments

- mechanical metamaterials



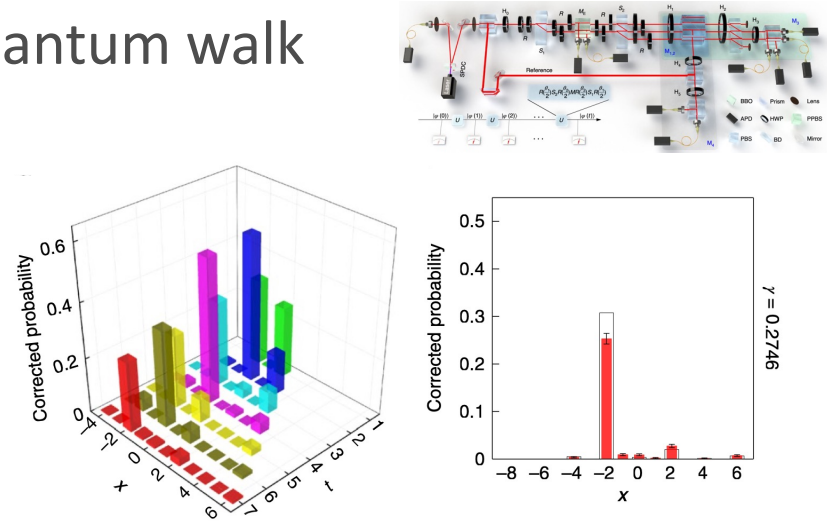
Brandenbourger *et al.*, Nat. Commun. **10**, 4608 (2019)

- electrical circuits



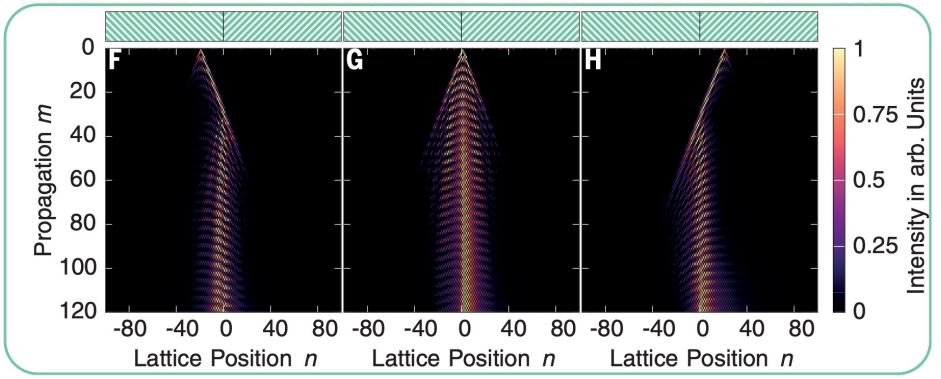
Helbig *et al.*, Nat. Phys. **16**, 747 (2020)

- quantum walk



Xiao *et al.*, Nat. Phys. **16**, 761 (2020)

- photonic lattice



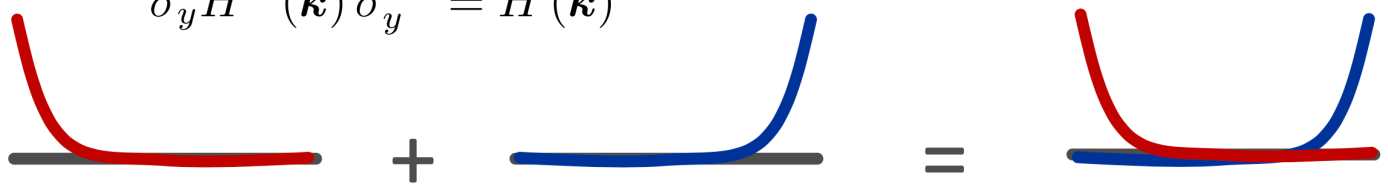
Weidemann *et al.*, Science **368**, 311 (2020)

Various skin effects

- Symmetry-protected skin effect

Okuma *et al.*, PRL **124**, 086801 (2020)

$$\sigma_y H^T(\boldsymbol{k})\,\sigma_y^{-1} = H(\boldsymbol{k})$$

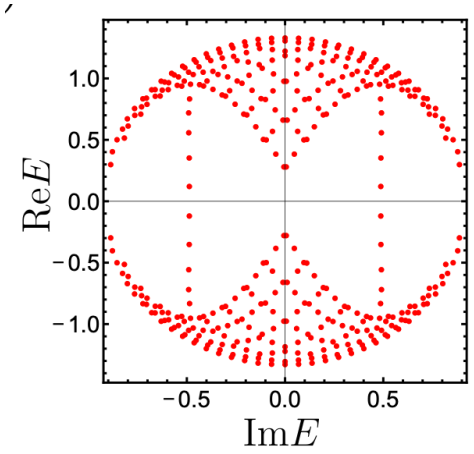


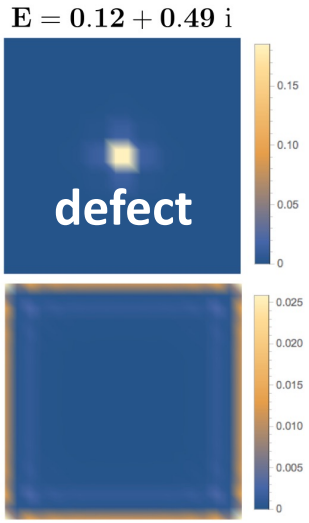
localized at left | localized at right | localized at both

reciprocal (Kramers) pair

reciprocal skin effect
Z_2 topological invariant

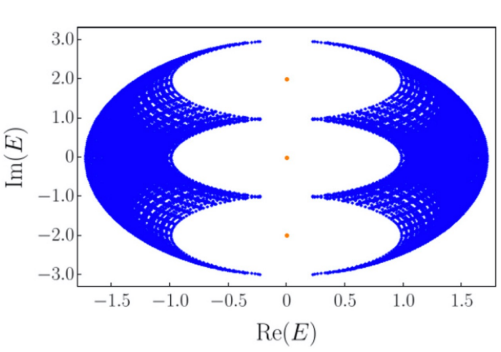
- 2D skin effect

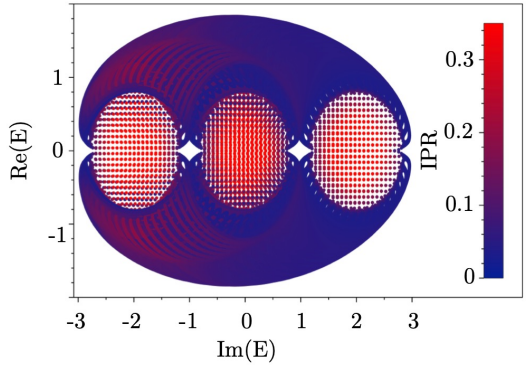




Okuma *et al.*, PRL **124**, 086801 (2020)

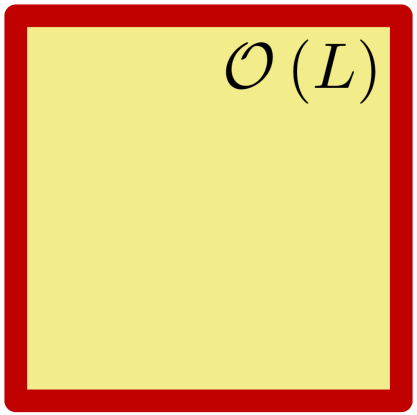
- 3D skin effect





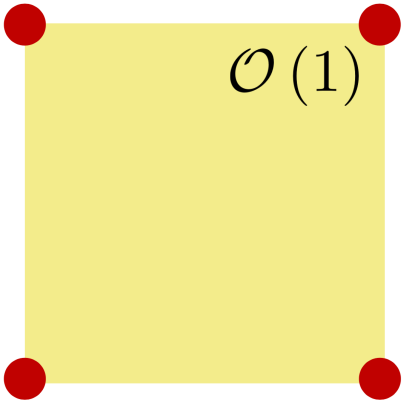
Terrier & Kunst, PRR **2**, 023364 (2020)
Denner *et al.*, arXiv:2008.01090

Hermitian, 1st



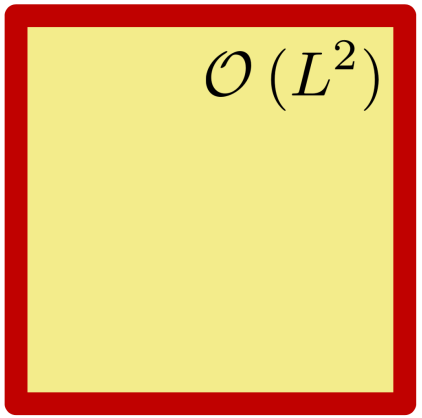
edge modes

Hermitian, 2nd



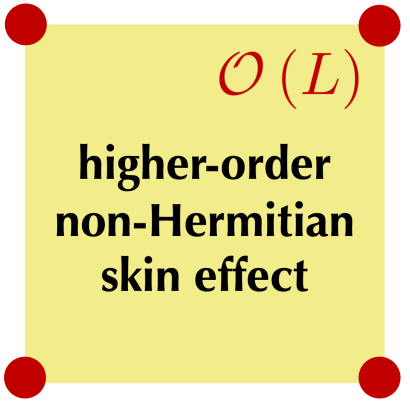
corner modes

Non-Hermitian, conventional



edge skin modes

Non-Hermitian, 2nd

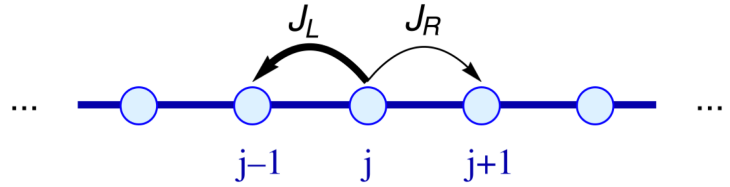


corner skin modes

Skin effect in one dimension

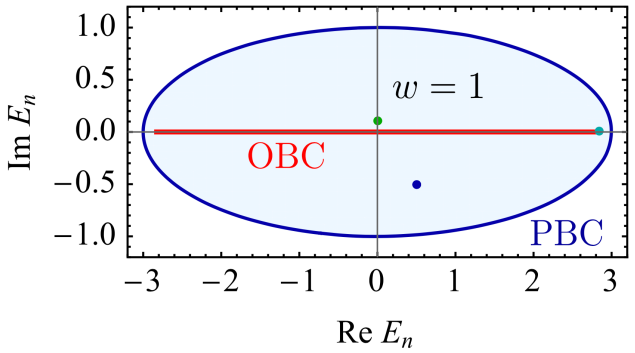
- Hatano-Nelson model

Hatano & Nelson, PRL **77**, 570 (1996)



$$\hat{H}_{\mathrm{HN}} = \sum_i \left(J_{\mathrm{R}} \hat{c}_{i+1}^{\dagger} \hat{c}_i + J_{\mathrm{L}} \hat{c}_i^{\dagger} \hat{c}_{i+1} \right)$$

$$H_{\mathrm{HN}}(k) = J_{\mathrm{R}} e^{\mathrm{i}k} + J_{\mathrm{L}} e^{-\mathrm{i}k}$$



Gong *et al.*, PRX **8**, 031079 (2018)

☆ Topological origin of the skin effect

$$W(E) := \oint \frac{dk}{2\pi \mathrm{i}} \frac{d}{dk} \log \det \left(H(k) - E \right)$$

intrinsic non-Hermitian topology

Zhang, Yang & Fang, arXiv:1910.01131

Okuma *et al.*, PRL **124**, 086801 (2020)

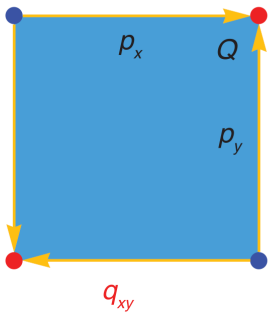
$$\tilde{H}(\boldsymbol{k}) = \begin{pmatrix} 0 & H_{\mathrm{HN}}(\boldsymbol{k}) \\ H_{\mathrm{HN}}^{\dagger}(\boldsymbol{k}) & 0 \end{pmatrix} = (J_{\mathrm{R}} + J_{\mathrm{L}})(\cos k)\,\tau_x - (J_{\mathrm{R}} - J_{\mathrm{L}})(\sin k)\,\tau_y$$

HN model (skin modes) ⟷ SSH model (zero modes)

Model

SSH model ⟷ HN model

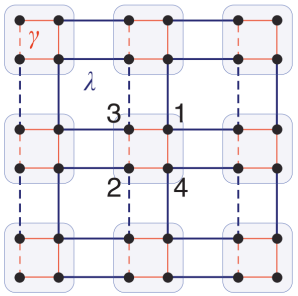
2nd-order TI (BBH model) ⟷ **2nd-order skin effect**



$$H_{\mathrm{BBH}}(\boldsymbol{k}) = (\gamma + \lambda\cos k_x)\,\tau_y + \lambda(\sin k_x)\,\sigma_z\tau_x + (\gamma + \lambda\cos k_y)\,\sigma_y\tau_x + \lambda(\sin k_y)\,\sigma_x\tau_x$$

⟷

$$H(\boldsymbol{k}) = -\mathrm{i}(\gamma + \lambda\cos k_x) + \lambda(\sin k_x)\,\sigma_z + (\gamma + \lambda\cos k_y)\,\sigma_y + \lambda(\sin k_y)\,\sigma_x$$



Benalcazar, Bernevig & Hughes, Science **357**, 61 (2017)

☆ Spatial symmetry

- inversion ⟶ $\sigma_y H(\boldsymbol{k})\,\sigma_y^{-1} = H(-\boldsymbol{k})$
- mirror ⟶ $\sigma_z H^{\dagger}(k_x, k_y)\,\sigma_z^{-1} = H(-k_x, k_y)$
- C_4 rotation ⟶ $-\mathrm{i}\sigma_y H^{\dagger}(k_x, k_y) = H(-k_y, k_x)$

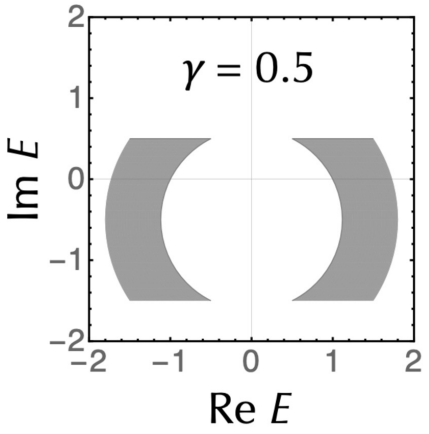
(unconventional symmetry unique to non-Hermitian systems)

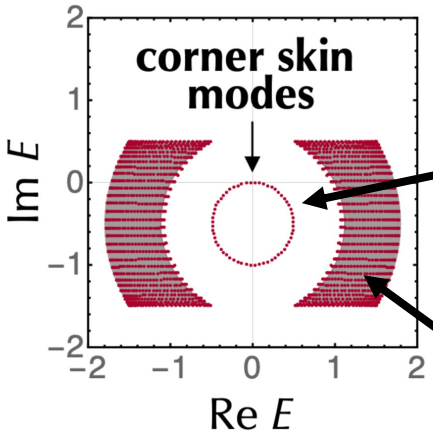
Second-order skin effect

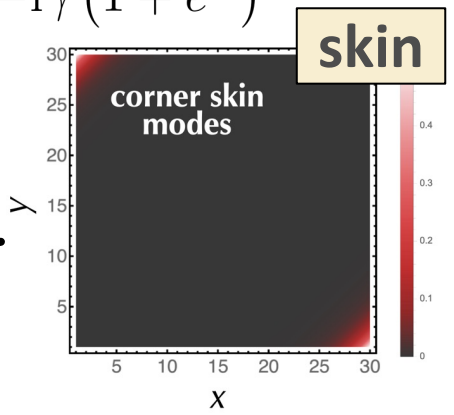
PBC

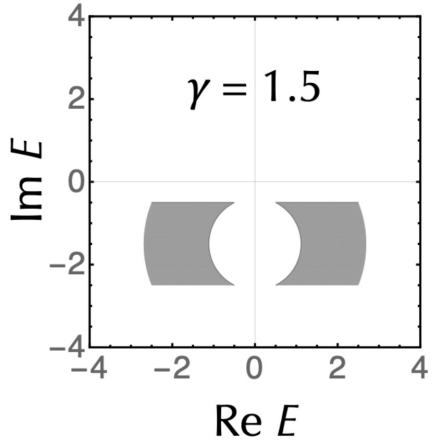
OBC

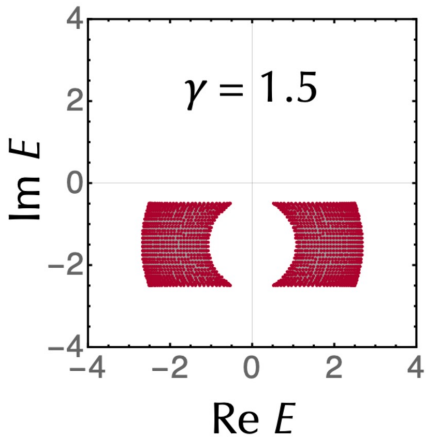
$$E = -\mathrm{i}\gamma\left(1 + e^{\mathrm{i}\theta}\right)$$

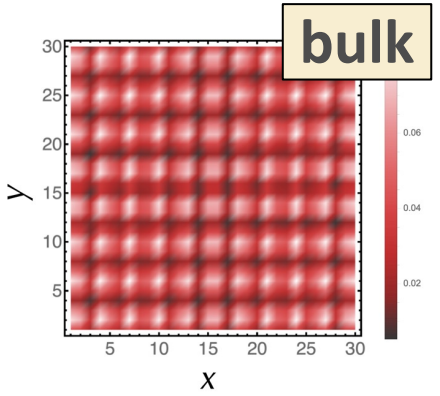












☆ The emergence of $O(L)$ corner skin modes.

Wess-Zumino term

☆ **C_4-rotation-type symmetry:** $U H^{\dagger}(k_x, k_y) V^{-1} = H(-k_y, k_x)$

$(U = -\mathrm{i}\sigma_y, V = 1)$

- C_4-rotation-type symmetry **vanishes the conventional skin effects**.

$$W_\mu := \oint_{\mathrm{BZ}} \frac{dk_\mu}{2\pi \mathrm{i}} \frac{\partial}{\partial k_\mu} \log\det\left[H(\boldsymbol{k})\right] = 0 \quad (\mu = x, y)$$

- C_4-rotation-type symmetry **quantizes the Wess-Zumino term**.

(not quantized in the absence of symmetry)

$$\mathrm{WZ}[H] := \frac{1}{24\pi^2} \int_{\mathrm{BZ}\times[0,1]} \mathrm{tr}\left[H^{-1} dH\right]^3 \in \{0, 1/2\}$$

Wess & Zumino, Phys. Lett. B **37**, 95 (1971)

☆ **The nontrivial WZ term is the origin of the corner skin effect.**

our model: $\mathrm{WZ}[H] = 1/2$ for $|\gamma/\lambda| < 1$

cf. zero modes at topological defects Teo & Kane, PRB **82**, 115120 (2010)

Non-Bloch band theory

☆ **The skin effect invalidates the Bloch band theory.**

⟶ **Non-Bloch band theory (Yao-Wang-Yokomizo-Murakami)**

$$|\beta_M| = |\beta_{M+1}|$$

$$|\beta_1| \leq \cdots \leq |\beta_{2M}| \text{ : solutions to } \det\left[H\left(\beta\right) - E\right] = 0$$

Yao & Wang, PRL **121**, 086803 (2018)
Yokomizo & Murakami, PRL **123**, 066404 (2019)

- **Symmetry can restore the Bloch band theory.** Kawabata *et al.*, PRX **9**, 041015 (2019)

e.g. inversion ⟶ $\beta_M = \beta_{M+1}^{-1}$ ⟶ (YWYM) $|\beta_M| = |\beta_{M+1}| = 1$

If the YWYM condition is valid even in higher dimensions,
the skin effect vanishes in the presence of inversion symmetry.

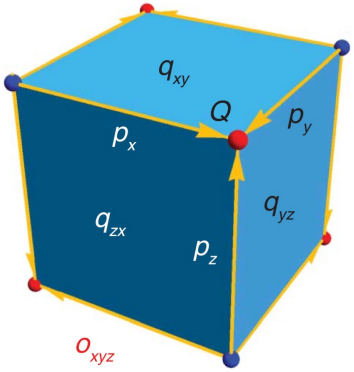
⟶ **But the *O* (*L*) corner skin modes emerge!**

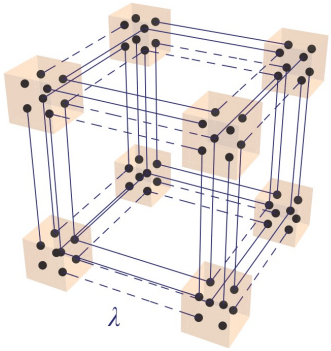
☆ **Breakdown of the YWYM condition in higher dimensions.**

[but expected to describe $O\left(L^d\right)$ modes]

Third-order skin effect

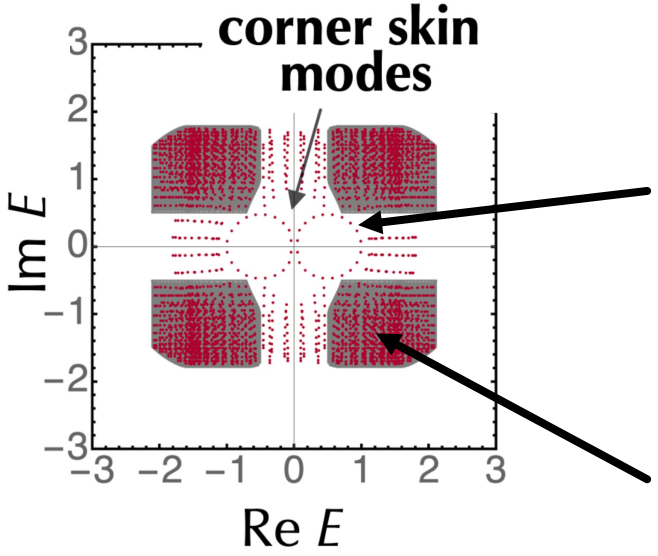
3rd-order TI (BBH model) ⟷ **3rd-order skin effect**

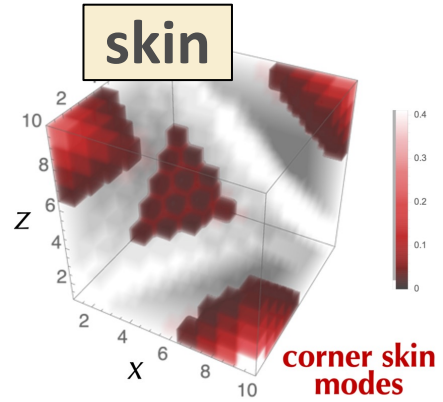


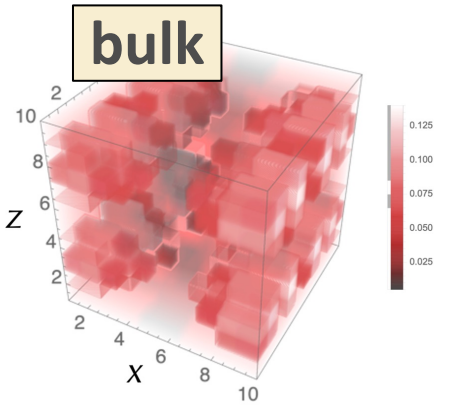


Benalcazar, Bernevig & Hughes, Science **357**, 61 (2017)

$$H(\boldsymbol{k}) = \mathrm{i}\lambda(\sin k_y)\,\sigma_x + \mathrm{i}(\gamma + \lambda\cos k_y)\,\sigma_y + \mathrm{i}\lambda(\sin k_x)\,\sigma_z + (\gamma + \lambda\cos k_x)\,\tau_z + \lambda(\sin k_z)\,\tau_y + (\gamma + \lambda\cos k_z)\,\tau_x,$$







Summary

arXiv:2008.07237

- We discover the higher-order non-Hermitian skin effect.
- 2nd-order skin effect in 2D: *O* (*L*) corner skin modes.
- 3rd-order skin effect in 3D: *O* (*L*) corner skin modes.
- Unique to non-Hermitian topological systems in higher dimensions.