Information Retrieval and Criticality in Parity-Time-Symmetric Systems

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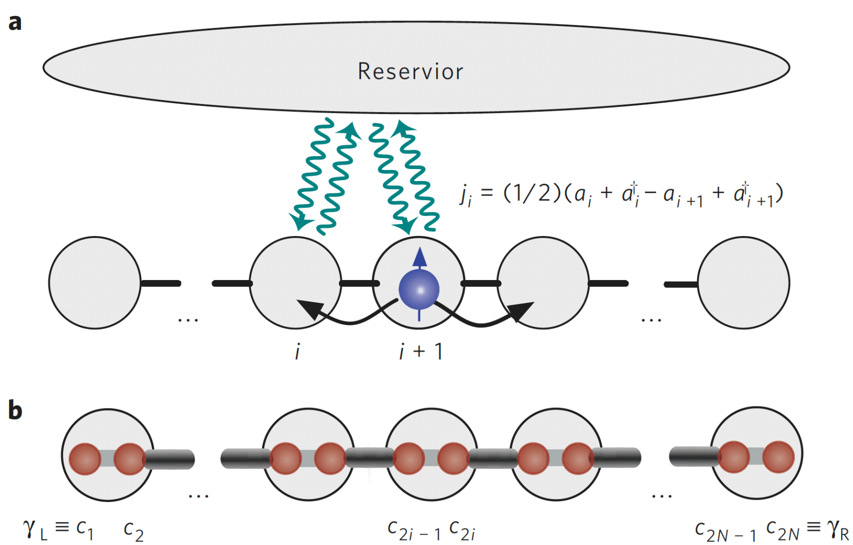
Quantum Control Using Dissipation

Loss is usually detrimental to the coherence of a system.

→ **However, loss can be useful in quantum control.**

e.g. **Dissipative** quantum computation and state engineering.

Verstraete *et al.*, Nat. Phys. **5**, 633 (2009).



e.g. Topological quantum computation (Majorana braiding) with **dissipation.**

Diehl *et al.*, Nat. Phys. **7**, 971 (2011).

Open System Having PT Symmetry

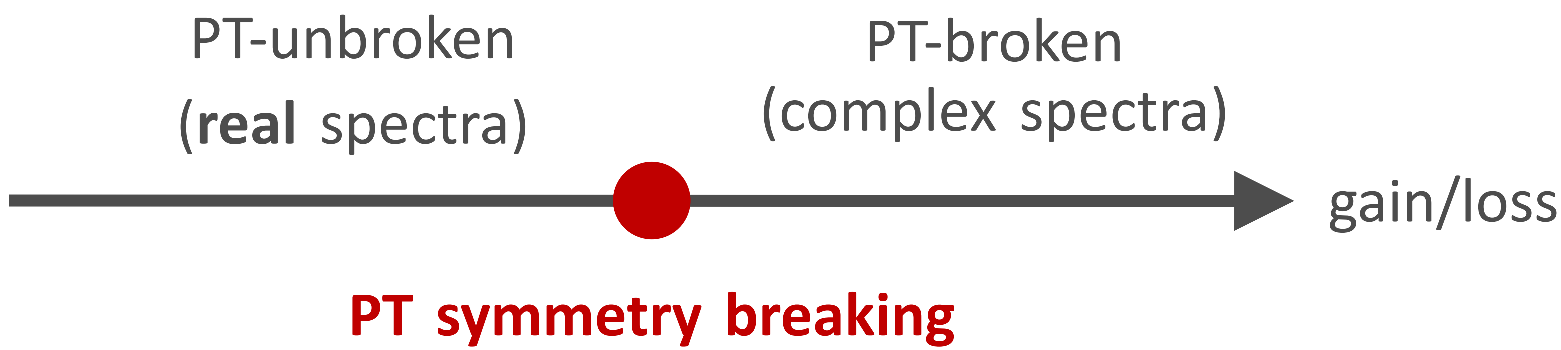
An open system **with balanced gain and loss** is effectively described by a **non-Hermitan Hamiltonian with PT symmetry**:

$$\hat{H}_{\mathrm{PT}}^{\dagger} \neq \hat{H}_{\mathrm{PT}} \quad \text{but} \quad [\hat{H}_{\mathrm{PT}}, \hat{\mathcal{P}}\hat{\mathcal{T}}] = 0$$

Bender and Boettcher, PRL **80**, 5243 (1998).

Non-unitary dynamics: $$\hat{\rho}(t) = \frac{e^{-\mathrm{i}\hat{H}_{\mathrm{PT}}t}\hat{\rho}_0 e^{\mathrm{i}\hat{H}_{\mathrm{PT}}^{\dagger}t}}{\mathrm{tr}\left[e^{-\mathrm{i}\hat{H}_{\mathrm{PT}}t}\hat{\rho}_0 e^{\mathrm{i}\hat{H}_{\mathrm{PT}}^{\dagger}t}\right]}$$

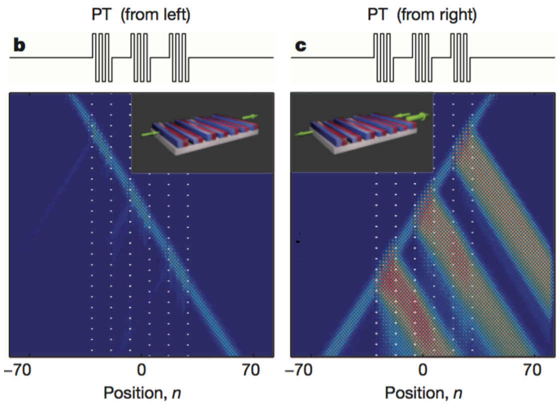
Brody and Graefe, PRL **109**, 230405 (2012).



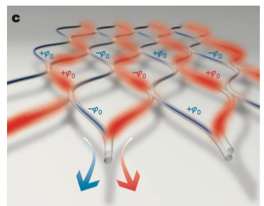
Experimental Realization

- **Synthetic photonic lattices**

Unidirectional light transport.



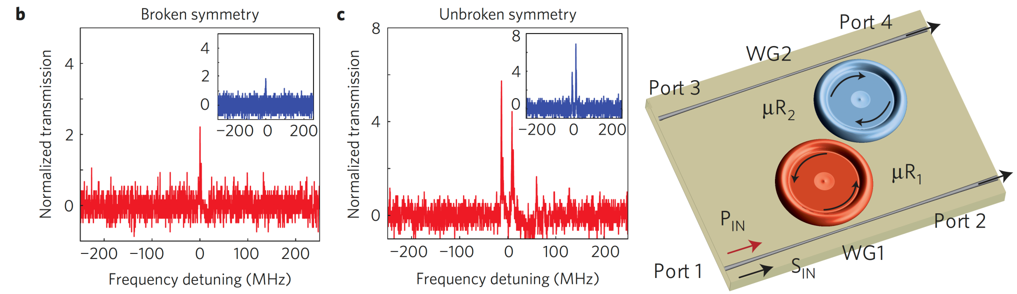
Regensburger *et al.*, Nature **488**, 167 (2012).



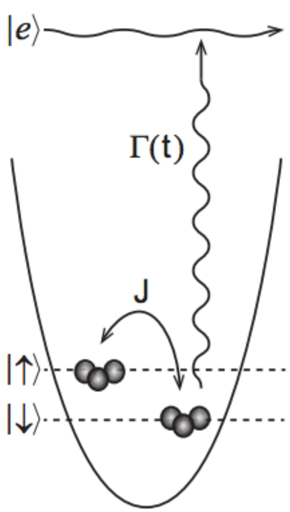
- **Microcavities**

Peng *et al.*, Nat. Phys. **10**, 394 (2014).

Reciprocal light transmission.



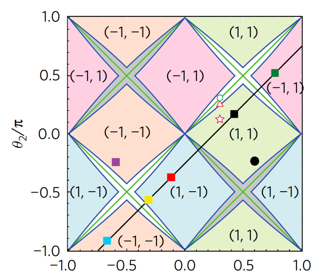
- **Ultracold atoms**



Stimulated Floquet dissipative Hamiltonian with PT symmetry.

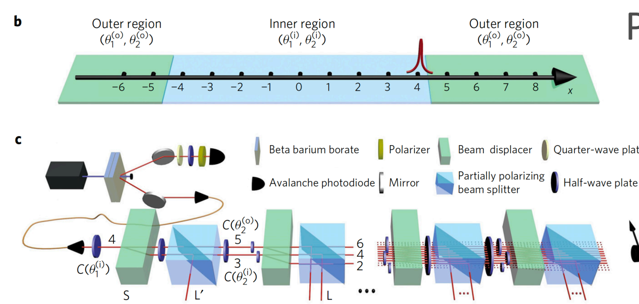
Li *et al.*, arXiv: 1608.05061.

- **Single-photon quantum walks**



Robust topological edge states due to PT symmetry.

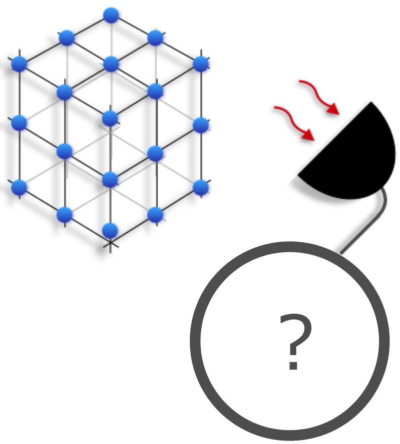
Xiao *et al.*, Nat. Phys. (in press).



Non-unitarity in Quantum Measurement

- **Quantum measurement**

System



Measuring apparatus

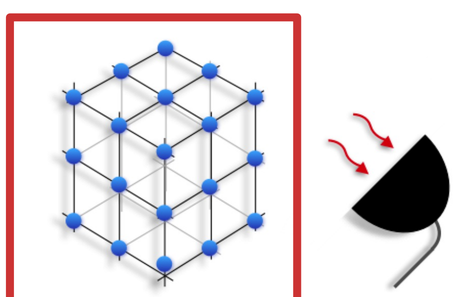
$$\hat{\rho}_i = \frac{\hat{M}_i \hat{\rho}_0 \hat{M}_i^\dagger}{\mathrm{tr}\left[\hat{M}_i \hat{\rho}_0 \hat{M}_i^\dagger\right]}$$

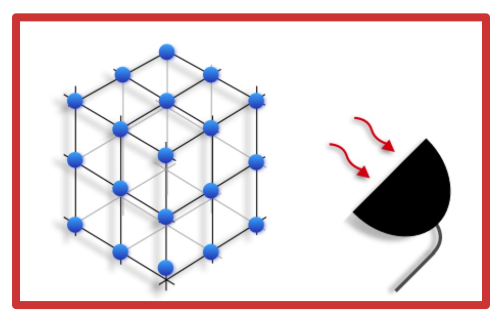
(1) Non-unitary state reduction.

→ **Information acquision of an observer**

(2) The entire system is Hermitian.

Non-unitary





Unitary

Questions

(1) How does information flow between a PT-symmetric system and its environment?

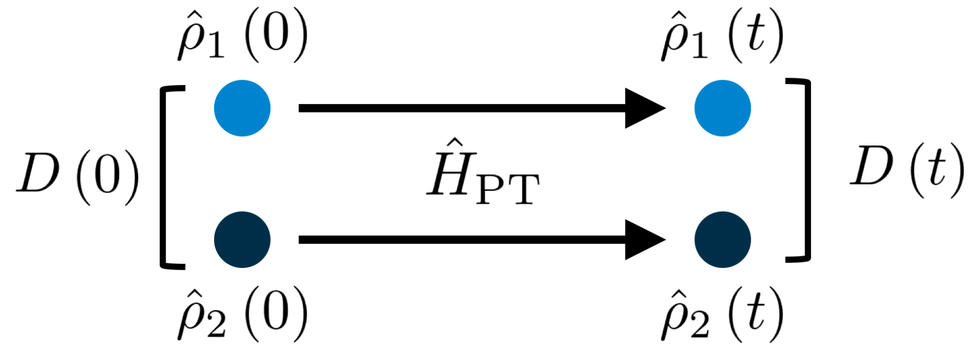
(2) Can a general PT-symmetric system be embedded into a larger Hermitian system?

Evaluation of Information Flow

- **Trace distance**

  (distinguishability)

$$D\left(t\right) := \frac{1}{2}\,\mathrm{tr}\,\left|\hat{\rho}_1\left(t\right) - \hat{\rho}_2\left(t\right)\right|$$



☆ **Contractive under a CPTP map $\mathcal{E}$**

(=a map without memory)

$$D\left(\mathcal{E}\hat{\rho}_1,\, \mathcal{E}\hat{\rho}_2\right) \leq D\left(\hat{\rho}_1,\, \hat{\rho}_2\right)$$

Breuer, Laine, and Piilo,
PRL **103**, 210401 (2009).

An increase in D

→ **Information backflow** from the environment to the system.

→ Presence of **memory effects! (Non-Markovian)**

Information Retrieval

We study the general behavior of the distinguishability D .

- **PT-unbroken phase**

  D oscillates and always returns to the initial value.

  = the system retrieves the information!

  = revival of quantum coherence!

- **PT-broken phase**

  D exponentially decays (NO retrieval): $D \sim e^{-t/\tau}$

Phase	Information	Memory
PT-unbroken	**Reversible**	**Non-Markovian**
PT-broken	**Irreversible**	Markovian

Universal Critical Behavior

Reversible (PT-unbroken) → Irreversible (PT-broken)

Criticality emerges around the transition point!

- PT-unbroken: recurrence time $T \sim |\Delta\lambda|^{-1/p} \quad (\Delta\lambda \to 0^-)$
- PT-broken: relaxation time $\tau \sim |\Delta\lambda|^{-1/p} \quad (\Delta\lambda \to 0^+)$
- PT transition: distinguishability itself $D \sim t^{-\delta(p)} \quad (\Delta\lambda = 0)$

($\Delta\lambda$: gain/loss parameter)

p : the order of the transition point

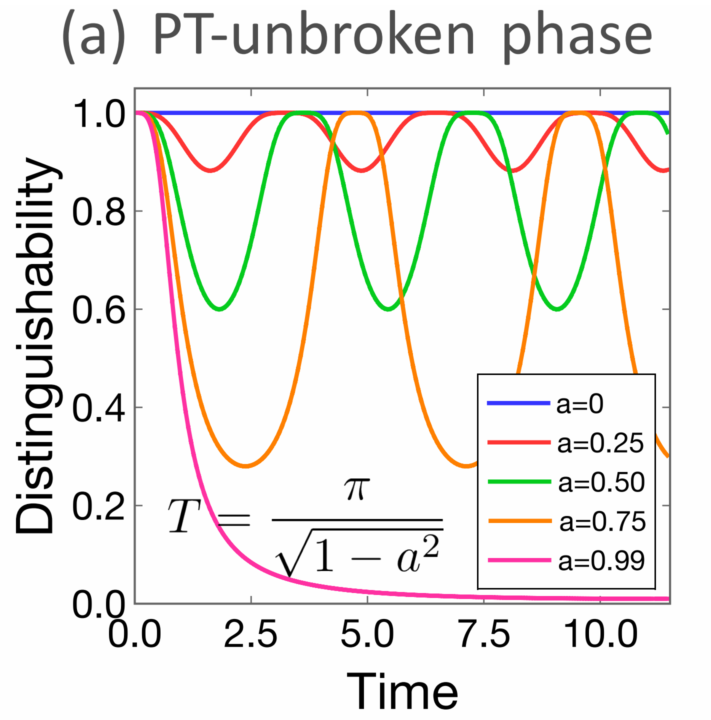
→ Determines the “PT universality class”

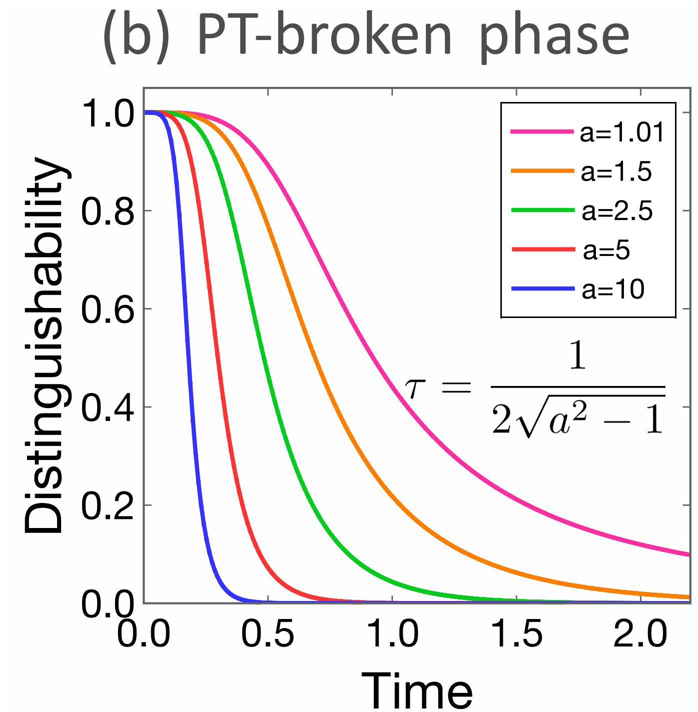
Example: Two-Level System

$$\hat{H}_{\mathrm{PT}} = \hat{\sigma}_x + \mathrm{i}a\,\hat{\sigma}_z$$

Initial states: $|\uparrow\rangle$ and $|\downarrow\rangle$

$$\rightarrow D(t) = \left[1 + \left(\frac{2a\sin^2\left(\sqrt{1-a^2}\,t\right)}{1-a^2}\right)^2\right]^{-1/2}$$





Embedding into a Larger Hermitian System

- **PT-unbroken phase**

Possible with a **finite** (two-level) ancilla:

$$|\Psi_{\mathrm{tot}}(t)\rangle := \underline{|\uparrow\rangle} \otimes |\psi_{\mathrm{PT}}(t)\rangle + |\downarrow\rangle \otimes \left(\underline{\hat{\zeta}^{1/2}} |\psi_{\mathrm{PT}}(t)\rangle\right)$$

two-level ancilla
(measuring apparatus)

Unbroken PT symmetry

The extended system is **closed (Hermitian)**:

$$|\Psi_{\mathrm{tot}}(t)\rangle = e^{-\mathrm{i}\hat{\mathrm{H}}_{\mathrm{tot}}t} |\Psi_{\mathrm{tot}}(0)\rangle$$

$\hat{\mathrm{H}}_{\mathrm{tot}}$ is Hermitian.

☆ **Information that has flowed into the environment is actually stored in the entanglement with a hidden ancilla.**

Information Retrieval Revisited

- **Proof of the information retrieval**

  The extended **finite** Hermitian system returns back to the original.

  → The PT-symmetric system also returns.

- **PT-broken phase**

  The above extension in the PT-unbroken phase breaks down, and information flow is irreversible.

  → **Infinite ancillas are required!**

  (Irreversibility can emerge in **infinite** Hermitian systems)

☆ **Physical origin of the information retrieval:**

Although the environment is in general infinite,
it is in effect finite as long as PT symmetry is unbroken.

Summary

arXiv: 1705.04628

- Studied Information flow between a PT-symmetric system and its environment.
- Information retrieval in the PT-unbroken phase.
- Unique criticality at the PT transition point.
- Origin of the information retrieval:

  Finite-dimensional environment induced by PT symmetry.

Phase	Spectra	Info.	Memory	Ancilla
Unbroken	Real	Reversible	Non-Markovian	Finite
Broken	Complex	Irreversible	Markovian	Infinite