

Chern insulator [Qi-Wu-Zhang model; PRB 74, 085308 (2006)]

```
In[*]:=  $\sigma_0 = \{\{1, 0\}, \{0, 1\}\};$   
         $\sigma_x = \{\{0, 1\}, \{1, 0\}\};$   
         $\sigma_y = \{\{0, -i\}, \{i, 0\}\};$   
         $\sigma_z = \{\{1, 0\}, \{0, -1\}\};$   
         $\sigma_p = \{\{0, 1\}, \{0, 0\}\};$   
         $\sigma_m = \{\{0, 0\}, \{1, 0\}\};$   
  
In[*]:= SetOptions[ListPlot,  
  [オプション… [リストプロット  
    Frame → True, Joined → False, FrameStyle → Thick, FrameTicksStyle → 30,  
    [枠 [真 [点の結合 [偽 [枠のスタイル [太い [枠目盛のスタイル  
    BaseStyle → {FontFamily → "Helvetica", FontSize → 20}, PlotStyle →  
    [ベーススタイル [フォントファミリ [フォントサイズ [プロットスタイル  
    {{RGBColor[173 / 255, 0, 45 / 255], PointSize[.025]}}, AspectRatio → 0.7];  
    [RGBカラー [ポイントサイズ [縦横比
```

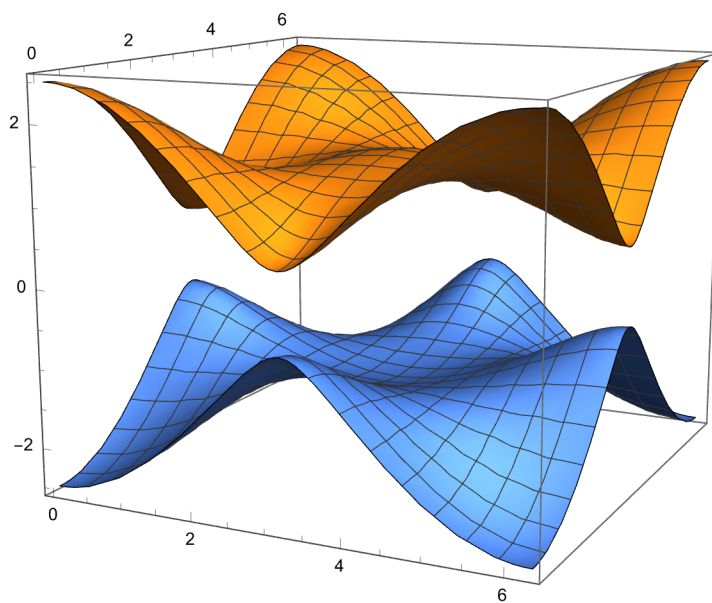
Bulk dispersion

```
In[*]:= t = 1.0;  
        m = -0.8;  
        H[kx_, ky_] := t * Sin[kx] *  $\sigma_x$  + t * Sin[ky] *  $\sigma_y$  + (m + t * Cos[kx] + t * Cos[ky]) *  $\sigma_z$ ;  
        [正弦 [正弦 [余弦 [余弦  
        Eval[kx_, ky_] :=  
        Sqrt[(t * Sin[kx])^2 + (t * Sin[ky])^2 + (m + t * Cos[kx] + t * Cos[ky])^2];  
        [平方根 [正弦 [正弦 [余弦 [余弦  
        Vecc[kx_, ky_] := Block[{hx, hy, hz, norm},  
        [ブロック  
        hx = t * Sin[kx];  
        [正弦  
        hy = t * Sin[ky];  
        [正弦  
        hz = m + t * Cos[kx] + t * Cos[ky];  
        [余弦 [余弦  
        norm = Sqrt[2 * Eval[kx, ky] * (hz + Eval[kx, ky])];  
        [平方根  
        (1 / norm) * {-hx + i * hy, hz + Eval[kx, ky]}  
        ]; (* occupied band: negative eigenenergy *)
```

m=0.5: topologically nontrivial gapped phase (C=-1)

```
In[ ]:= Plot3D[{Eval[kx, ky], -Eval[kx, ky]},  
[3Dプロット  
{kx, 0, 2  $\pi$ }, {ky, 0, 2  $\pi$ }, BoxRatios  $\rightarrow$  Automatic]  
[境界ボックス… [自動
```

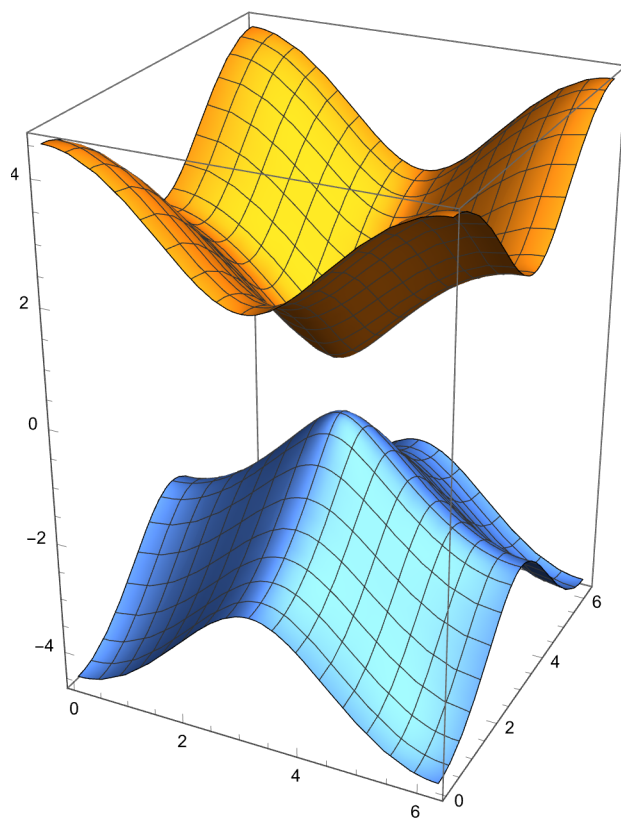
Out[]=



$m = 2.5$: topologically trivial gapped phase ($C=0$)

```
In[ ]:= Plot3D[{Eval[kx, ky], -Eval[kx, ky]},  
[3Dプロット  
{kx, 0, 2  $\pi$ }, {ky, 0, 2  $\pi$ }, BoxRatios  $\rightarrow$  Automatic]  
[境界ボックス… [自動
```

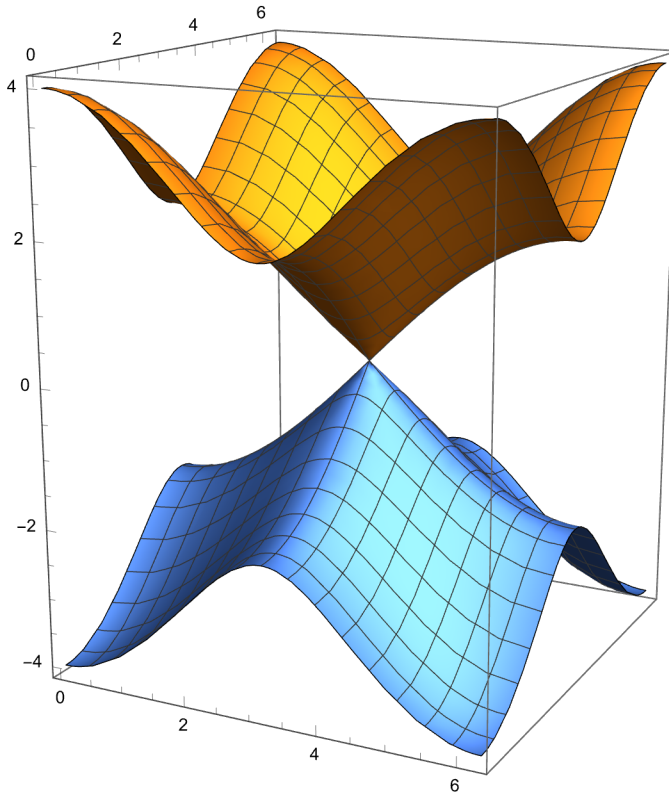
Out[]=



$m = 2.0$: gapless phase

```
In[ ]:= Plot3D[{Eval[kx, ky], -Eval[kx, ky]},
[3Dプロット
{kx, 0, 2  $\pi$ }, {ky, 0, 2  $\pi$ }, BoxRatios  $\rightarrow$  Automatic]
[境界ボックス… [自動
```

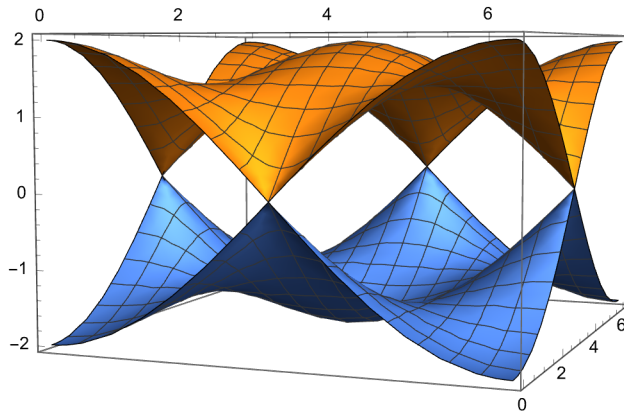
Out[]:=



$m = 0.0$: gapless phase

```
In[ ]:= Plot3D[{Eval[kx, ky], -Eval[kx, ky]},
[3Dプロット
{kx, 0, 2  $\pi$ }, {ky, 0, 2  $\pi$ }, BoxRatios  $\rightarrow$  Automatic]
[境界ボックス… [自動
```

Out[]:=



Chern number: pump of polarization

```

In[*]:= kmesh = 50;
         $\delta k = 2 \pi / \text{kmesh} * 0.1$ ;
        Utab =
          Table[Evec[kx, ky], {kx,  $\delta k$ ,  $2 \pi + \delta k$ ,  $2 \pi / \text{kmesh}$ }, {ky,  $\delta k$ ,  $2 \pi + \delta k$ ,  $2 \pi / \text{kmesh}$ }]
          ↳リストを作成
        Pxtab = Table[{ $\delta k + y * 2 \pi / \text{kmesh}$ ,
          ↳リストを作成
            Arg[Product[Utab[[x, y]]†.Utab[[x + 1, y]], {x, 1, kmesh}]] / (2  $\pi$ )}, {y, 1, kmesh}];
          ↳偏角 ↳乗積

```

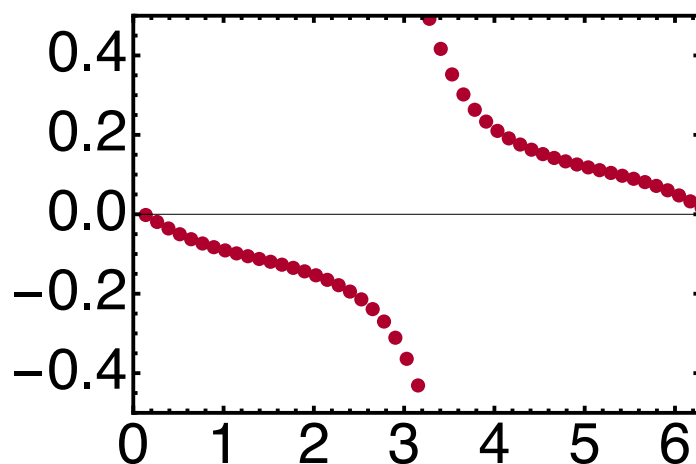
$m = 0.5$: topologically nontrivial gapped phase ($C = -1$)

```

In[*]:= ListPlot[Pxtab, PlotRange → {{0, 2  $\pi$ }, 0.5 {-1, 1}}]
        ↳リストプロット ↳プロット範囲

```

Out[*]=



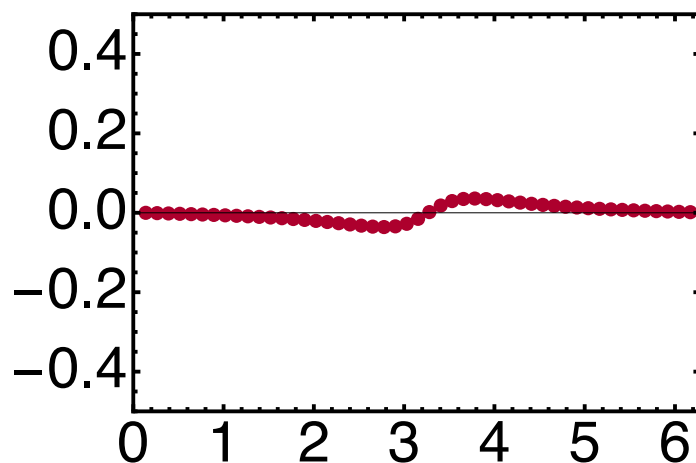
$m = 2.5$: topologically trivial gapped phase ($C = 0$)

```

In[*]:= ListPlot[Pxtab, PlotRange → {{0, 2  $\pi$ }, 0.5 {-1, 1}}]
        ↳リストプロット ↳プロット範囲

```

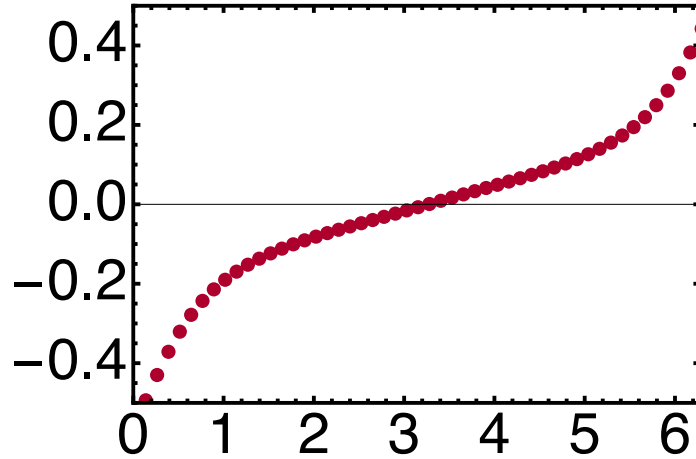
Out[*]=



$m = -0.8$: topologically nontrivial gapped phase ($C = +1$)

```
In[*]:= ListPlot[Pxstab, PlotRange -> {{0, 2  $\pi$ }, 0.5 {-1, 1}}]
[リストプロット] [プロット範囲]
```

Out[*]=



Chern number in the discrete Brillouin zone [Fukui-Hatsugai-Suzuki, JPSJ 74, 1674 (2005)]

```
In[*]:= Sum[-Arg[Utab[[kx, ky]]†.Utab[[kx + 1, ky]] * Utab[[kx + 1, ky]]†.Utab[[kx + 1, ky + 1]] *
[総和] [偏角]
    Utab[[kx + 1, ky + 1]]†.Utab[[kx, ky + 1]] * Utab[[kx, ky + 1]]†.Utab[[kx, ky]]],
    {kx, 1, kmesh}, {ky, 1, kmesh}] / (2  $\pi$ ) // Chop
[近い数にする]
```

Out[*]=

1.

Spectrum under the open boundary conditions (x: OBC, y: PBC)

```

σ0 = {{1, 0}, {0, 1}};
σx = {{0, 1}, {1, 0}};
σy = {{0, -i}, {i, 0}};
σz = {{1, 0}, {0, -1}};
σp = {{0, 1}, {0, 0}};
σm = {{0, 0}, {1, 0}};
ChainLength = 30;
CMat1 = SparseArray[{{Band[{1, 1]} → 1}], {ChainLength, ChainLength}};
           ↳疎な配列           ↳帯対角行列
CMat2 = SparseArray[{{Band[{1, 2]} → 1}], {ChainLength, ChainLength}};
           ↳疎な配列           ↳帯対角行列
CMat3 = SparseArray[{{Band[{2, 1]} → 1}], {ChainLength, ChainLength}};
           ↳疎な配列           ↳帯対角行列

t = 1.0;
m = -0.8;
Ly = 100;
kList = Table[2 π * n / Ly, {n, 0, Ly - 1}];
           ↳リストを作成

Mass[ky_] := t * Sin[ky] * σy + (m + t * Cos[ky]) * σz;
           ↳正弦           ↳余弦

Hop = t * (-i / 2) * σx + t * (1 / 2) * σz;
Hx[ky_] := KroneckerProduct[CMat1, Mass[ky]] +
           ↳クロネッカー積
           KroneckerProduct[CMat2, Hop] + KroneckerProduct[CMat3, Hop];
           ↳クロネッカー積           ↳クロネッカー積

In[*]:= EvalList[ky_] := Chop[Eigenvalues[Hx[ky]]];
           ↳近…   ↳固有値

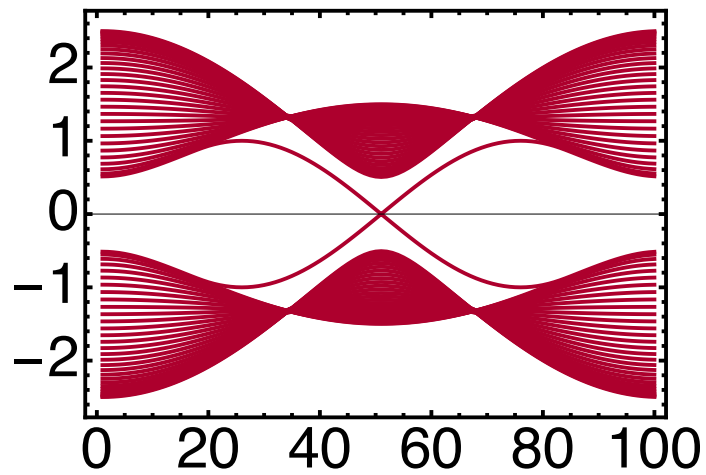
EvalData = Table[EvalList[kList[[n]]], {n, 1, Ly}];
           ↳リストを作成

```

$m = 0.5$: topologically nontrivial gapped phase ($C = -1$)

```
In[ ]:= ListPlot[Transpose[Sort /@ EvalData], Joined → True]
      ↳リスト…   ↳転置       ↳並べ替え       ↳点の結合   ↳真
```

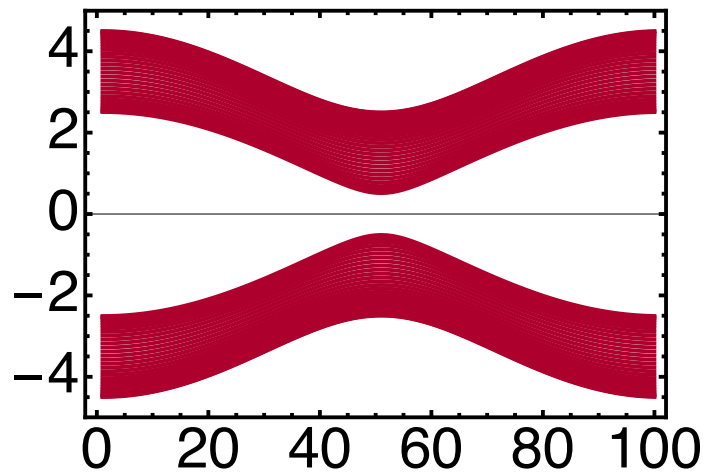
Out[]:=



$m = 2.5$: topologically trivial gapped phase ($C = 0$)

```
In[ ]:= ListPlot[Transpose[Sort /@ EvalData], Joined → True]
      ↳リスト…   ↳転置       ↳並べ替え       ↳点の結合   ↳真
```

Out[]:=



$m = -0.8$: topologically nontrivial gapped phase ($C = +1$)

```
In[*]:= ListPlot[Transpose[Sort /@ EvalData], Joined → True]
```

└リスト… └転置 └並べ替え └点の結合 └真

Out[*]=

