



東京大学 物性研究所

THE INSTITUTE FOR SOLID STATE PHYSICS
THE UNIVERSITY OF TOKYO

Complex class: Z topological invariants

Kohei Kawabata

(Institute for Solid State Physics, University of Tokyo)

Classification of topological insulators

Periodic table of topological insulators and superconductors

AZ Symmetry				Dimension							
Class	TRS	PHS	CS	0	1	2	3	4	5	6	7
A	0	0	0	\mathbb{Z}	0	\mathbb{Z}	0	\mathbb{Z}	0	\mathbb{Z}	0
AIII	0	0	1	0	\mathbb{Z}	0	\mathbb{Z}	0	\mathbb{Z}	0	\mathbb{Z}
AI	+1	0	0	\mathbb{Z}	0	0	0	$2\mathbb{Z}$	0	\mathbb{Z}_2	\mathbb{Z}_2
BDI	+1	+1	1	\mathbb{Z}_2	\mathbb{Z}	0	0	0	$2\mathbb{Z}$	0	\mathbb{Z}_2
D	0	+1	0	\mathbb{Z}_2	\mathbb{Z}_2	\mathbb{Z}	0	0	0	$2\mathbb{Z}$	0
DIII	-1	+1	1	0	\mathbb{Z}_2	\mathbb{Z}_2	\mathbb{Z}	0	0	0	$2\mathbb{Z}$
AII	-1	0	0	$2\mathbb{Z}$	0	\mathbb{Z}_2	\mathbb{Z}_2	\mathbb{Z}	0	0	0
CII	-1	-1	1	0	$2\mathbb{Z}$	0	\mathbb{Z}_2	\mathbb{Z}_2	\mathbb{Z}	0	0
C	0	-1	0	0	0	$2\mathbb{Z}$	0	\mathbb{Z}_2	\mathbb{Z}_2	\mathbb{Z}	0
CI	+1	-1	1	0	0	0	$2\mathbb{Z}$	0	\mathbb{Z}_2	\mathbb{Z}_2	\mathbb{Z}

Schnyder, Ryu, Furusaki & Ludwig, PRB **78**, 195125 (2008)

Kitaev, AIP Conf. Proc. **1134**, 22 (2009)

Ryu, Schnyder, Furusaki & Ludwig, NJP **12**, 065010 (2010)

Classification of topological insulators

Periodic table of topological insulators and superconductors

AZ Symmetry				Dimensional Anomalous Anomalous							
Class	TRS	PHS	CS	Chern number: quantum Hall insulator							
A	0	0	0	\mathbb{Z}	0	\mathbb{Z}	0	\mathbb{Z}	0	\mathbb{Z}	0
AIII	0	0	1	0	\mathbb{Z}	0	\mathbb{Z}	0	\mathbb{Z}	0	\mathbb{Z}
AI	+1	0	0	winding number: SSH model							
BDI	+1	+1	1	\mathbb{Z}_2	\mathbb{Z}	\mathbb{Z}_2	\mathbb{Z}	\mathbb{Z}_2	\mathbb{Z}	\mathbb{Z}_2	\mathbb{Z}
D	0	+1	0	\mathbb{Z}_2	\mathbb{Z}_2	\mathbb{Z}	0	0	0	$2\mathbb{Z}$	0
DIII	-1	+1	1	0	\mathbb{Z}_2	\mathbb{Z}_2	\mathbb{Z}	0	0	0	$2\mathbb{Z}$
AII	-1	0	0	$2\mathbb{Z}$	0	\mathbb{Z}_2	\mathbb{Z}_2	\mathbb{Z}	0	0	0
CII	-1	-1	1	0	$2\mathbb{Z}$	0	\mathbb{Z}_2	\mathbb{Z}_2	\mathbb{Z}	0	0
C	0	-1	0	0	0	$2\mathbb{Z}$	0	\mathbb{Z}_2	\mathbb{Z}_2	\mathbb{Z}	0
CI	+1	-1	1	0	0	0	$2\mathbb{Z}$	0	\mathbb{Z}_2	\mathbb{Z}_2	\mathbb{Z}

Schnyder, Ryu, Furusaki & Ludwig, PRB **78**, 195125 (2008)

Kitaev, AIP Conf. Proc. **1134**, 22 (2009)

Ryu, Schnyder, Furusaki & Ludwig, NJP **12**, 065010 (2010)

Z topological invariants

☆ Z-classified topological phases in the tenfold periodic table

- Even spatial dimensions $d = 0, 2, 4, \dots$ with no symmetry (class A)

$d = 0$: 0th Chern number

$d = 2$: 1st Chern number \leftarrow quantum Hall effect

$d = 4$: 2nd Chern number \leftarrow 4D quantum Hall effect

- Odd spatial dimensions $d = 1, 3, 5, \dots$ with chiral symmetry (class AIII)

$d = 1$: 1D winding number \leftarrow SSH model

$d = 3$: 3D winding number \leftarrow 3D topological insulators/superconductors

$d = 5$: 5D winding number

Classification of topological insulators

Periodic table of topological insulators and superconductors

AZ Symmetry				Dimension $(d/2)$ th Chern number							
Class	TRS	PHS	CS	0	1	2	3	4	5	6	7
A	0	0	0	\mathbb{Z}	0	\mathbb{Z}	0	\mathbb{Z}	0	\mathbb{Z}	0
AIII	0	0	1	0	\mathbb{Z}	0	\mathbb{Z}	0	\mathbb{Z}	0	\mathbb{Z}
AI	+1	0	0	\mathbb{Z}	0	0	0	$2\mathbb{Z}$	0	\mathbb{Z}_2	\mathbb{Z}_2
BDI	+1	+1	1	\mathbb{Z}_2	\mathbb{Z}	0	0	0	$2\mathbb{Z}$	0	\mathbb{Z}_2
D	0	+1	0	\mathbb{Z}_2	\mathbb{Z}_2	\mathbb{Z}	0	0	0	$2\mathbb{Z}$	0
DIII	-1	+1	1	0	\mathbb{Z}_2	\mathbb{Z}_2	\mathbb{Z}	0	0	0	$2\mathbb{Z}$
AII	-1	0	0	$2\mathbb{Z}$	0	\mathbb{Z}_2	\mathbb{Z}_2	\mathbb{Z}	0	0	0
CII	-1	-1	1	0	$2\mathbb{Z}$	0	\mathbb{Z}_2	\mathbb{Z}_2	\mathbb{Z}	0	0
C	0	-1	0	0	0	$2\mathbb{Z}$	0	\mathbb{Z}_2	\mathbb{Z}_2	\mathbb{Z}	0
CI	+1	-1	1	0	0	0	$2\mathbb{Z}$	0	\mathbb{Z}_2	\mathbb{Z}_2	\mathbb{Z}

dD winding number

Schnyder, Ryu, Furusaki & Ludwig, PRB **78**, 195125 (2008)

Kitaev, AIP Conf. Proc. **1134**, 22 (2009)

Ryu, Schnyder, Furusaki & Ludwig, NJP **12**, 065010 (2010)

Z topological invariants

- **Complex class (classes A & AIII)**

Only **unitary symmetry** is relevant.

- **Real class (classes AI, BDI, D, DIII, AII, CII, C, & CI)**

Antiunitary symmetry is relevant (time-reversal & particle-hole symmetry).

Certain Z topology **survives**, but other Z topology **vanishes**.

Antiunitary symmetry can protect new types of topological phases characterized by **Z_2 topological invariants!**

cf. Spatial symmetry can lead to Z_2 topological phases even if it is unitary.

Classification of topological insulators

Periodic table of topological insulators and superconductors

AZ Symmetry				Dimension							
Class	TRS	PHS	CS	0	1	2	3	4	5	6	7
A	0	0	0	\mathbb{Z}	0	\mathbb{Z}	Chern number				0
AIII	0	0	1	0	\mathbb{Z}	0	\mathbb{Z}	0	\mathbb{Z}	0	\mathbb{Z}
AI	+1	0	0	\mathbb{Z}	0	0	0	$2\mathbb{Z}$	0	\mathbb{Z}_2	\mathbb{Z}_2
BDI	+1	+1	1	\mathbb{Z}_2	\mathbb{Z}	0	0	0	$2\mathbb{Z}$	0	\mathbb{Z}_2
D	0	+1	0	\mathbb{Z}_2	\mathbb{Z}_2	\mathbb{Z}	0	0	0	$2\mathbb{Z}$	0
DIII	-1	+1	1	0	\mathbb{Z}_2	\mathbb{Z}_2	\mathbb{Z}	0	0	0	$2\mathbb{Z}$
AII	-1	0	0	$2\mathbb{Z}$	0	\mathbb{Z}_2	\mathbb{Z}_2	\mathbb{Z}	0	0	0
CII	-1	-1	1	0	$2\mathbb{Z}$	0	\mathbb{Z}_2	\mathbb{Z}_2	\mathbb{Z}	0	0
C	0	-1	0	0	0	0	0	0	0	0	0
CI	+1	-1	0	0	0	0	0	0	0	0	0

TRS vanishes the Chern number, but leads to a new \mathbb{Z}_2 topological invariant!

Schnyder, Ryu, Furusaki & Ludwig, PRB **78**, 195125 (2008)

Kitaev, AIP Conf. Proc. **1134**, 22 (2009)

Ryu, Schnyder, Furusaki & Ludwig, NJP **12**, 065010 (2010)