Homework I (Condensed Matter Physics II)

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Let us consider the Su-Schrieffer-Heeger model with the additional hopping. The Hamiltonian reads

$$\hat{H} = \sum_{n=1}^{L} \left[v \left(\hat{b}_n^\dagger \hat{a}_n + \hat{a}_n^\dagger \hat{b}_n \right) + t \left(\hat{a}_{n+1}^\dagger \hat{b}_n + \hat{b}_n^\dagger \hat{a}_{n+1} \right) + g \left(\hat{a}_{n+2}^\dagger \hat{b}_n + \hat{b}_n^\dagger \hat{a}_{n+2} \right) \right], \tag{1}$$

where $\hat{a}_n$ and $\hat{b}_n$ ($\hat{a}_n^\dagger$ and $\hat{b}_n^\dagger$) annihilate (create) fermions on the two sublattices. For simplicity, we assume $v, t, g > 0$.

(1) Perform the Fourier transform and write down the Bloch Hamiltonian $H(k)$. Confirm that the obtained Bloch Hamiltonian $H(k)$ respects chiral symmetry.

(2) Calculate the energy dispersion $E(k)$ and identify the gap-closing condition.

(3) In each gapped phase, calculate the topological invariant (i.e., winding number). Then, make a phase diagram with respect to v/t and g/t.

(4) (optional) Using numerical calculations, confirm the emergence of zero modes under the open boundary conditions consistent with the bulk topological invariant.