

# Z2 topological insulator [(generalized) Bernevig-Hughes- Zhang model; Science 314, 1757 (2006)]

```
In[*]:= SetOptions[ListPlot,  
  [オプション… [リストプロット  
    Frame → True, Joined → False, FrameStyle → Thick, FrameTicksStyle → 30,  
    [枠 [真 [点の結合 [偽 [枠のスタイル [太い [枠目盛のスタイル  
    BaseStyle → {FontFamily → "Helvetica", FontSize → 20}, PlotStyle →  
    [ベーススタイル [フォントファミリー [フォントサイズ [プロットスタイル  
    {{RGBColor[173 / 255, 0, 45 / 255], PointSize[.025]}}, AspectRatio → 0.7];  
    [RGBカラー [ポイントサイズ [縦横比
```

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Spectrum under the open boundary conditions (x: OBC,  
y: PBC)

```
In[*]:=  $\sigma_0 = \{\{1, 0\}, \{0, 1\}\};$   
 $\sigma_x = \{\{0, 1\}, \{1, 0\}\};$   
 $\sigma_y = \{\{0, -i\}, \{i, 0\}\};$   
 $\sigma_z = \{\{1, 0\}, \{0, -1\}\};$   
 $\sigma_p = \{\{0, 1\}, \{0, 0\}\};$   
 $\sigma_m = \{\{0, 0\}, \{1, 0\}\};$   
 $\sigma_{00} = \text{KroneckerProduct}[\sigma_0, \sigma_0];$   
  [クロネッカー積  
 $\sigma_{0x} = \text{KroneckerProduct}[\sigma_0, \sigma_x];$   
  [クロネッカー積  
 $\sigma_{0y} = \text{KroneckerProduct}[\sigma_0, \sigma_y];$   
  [クロネッカー積  
 $\sigma_{0z} = \text{KroneckerProduct}[\sigma_0, \sigma_z];$   
  [クロネッカー積  
 $\sigma_{x0} = \text{KroneckerProduct}[\sigma_x, \sigma_0];$   
  [クロネッカー積  
 $\sigma_{xx} = \text{KroneckerProduct}[\sigma_x, \sigma_x];$   
  [クロネッカー積  
 $\sigma_{xy} = \text{KroneckerProduct}[\sigma_x, \sigma_y];$   
  [クロネッカー積  
 $\sigma_{xz} = \text{KroneckerProduct}[\sigma_x, \sigma_z];$   
  [クロネッカー積
```

```

σy0 = KroneckerProduct[σy, σ0];
      ↳クロネッカー積
σyx = KroneckerProduct[σy, σx];
      ↳クロネッカー積
σyy = KroneckerProduct[σy, σy];
      ↳クロネッカー積
σyz = KroneckerProduct[σy, σz];
      ↳クロネッカー積
σz0 = KroneckerProduct[σz, σ0];
      ↳クロネッカー積
σzx = KroneckerProduct[σz, σx];
      ↳クロネッカー積
σzy = KroneckerProduct[σz, σy];
      ↳クロネッカー積
σzz = KroneckerProduct[σz, σz];
      ↳クロネッカー積

ChainLength = 30;
CMat1 = SparseArray[{Band[{1, 1}] → 1}, {ChainLength, ChainLength}];
      ↳疎な配列      ↳帯対角行列
CMat2 = SparseArray[{Band[{1, 2}] → 1}, {ChainLength, ChainLength}];
      ↳疎な配列      ↳帯対角行列
CMat3 = SparseArray[{Band[{2, 1}] → 1}, {ChainLength, ChainLength}];
      ↳疎な配列      ↳帯対角行列

t = 1.0;
m = 0.5;
δy = 0.3; (* σxy *)
δz = 0.2; (* σxz *)
Ly = 100;
kList = Table[2 π * n / Ly, {n, 0, Ly - 1}];
      ↳リストを作成

Mass[ky_] := t * Sin[ky] * σ0y + (m + t * Cos[ky]) * σ0z + δy * σxy + δz * σxz;
      ↳正弦      ↳余弦

Hop = t * (-i / 2) * σzx + t * (1 / 2) * σ0z;
Hx[ky_] := KroneckerProduct[CMat1, Mass[ky]] +
      ↳クロネッカー積
      KroneckerProduct[CMat2, Hop] + KroneckerProduct[CMat3, Hop];
      ↳クロネッカー積      ↳クロネッカー積

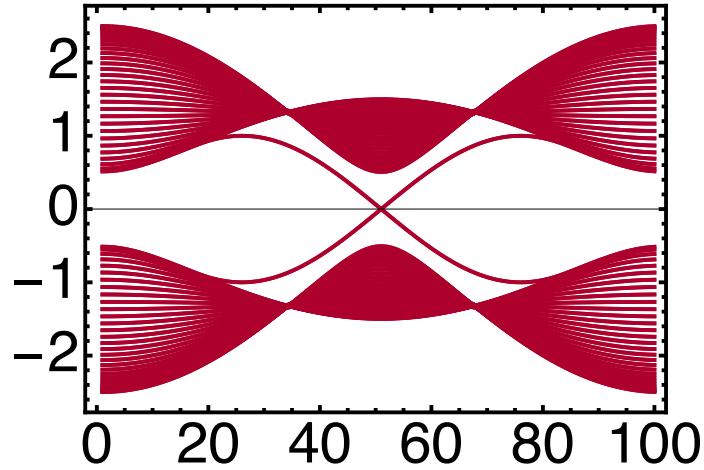
In[*]:= EvalList[ky_] := Chop[Eigenvalues[Hx[ky]]];
      ↳近…      ↳固有値

EvalData = Table[EvalList[kList[[n]]], {n, 1, Ly}];
      ↳リストを作成

```

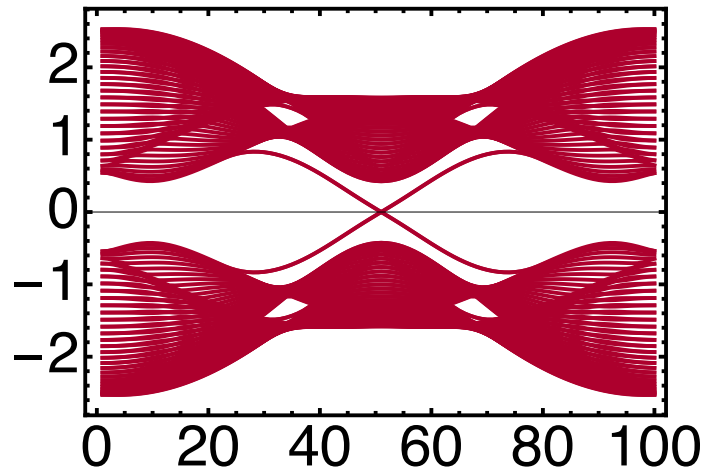
$m = 0.5$  &  $\delta y = 0.0$  : topologically nontrivial gapped phase ( $\delta z = 0.0$ )

```
In[ ]:= ListPlot[Transpose[Sort /@ EvalData], Joined → True]
      リスト…  転置  並べ替え  点の結合  真
Out[ ]:=
```



$m = 0.5$  &  $\delta y = 0.3$  : topologically nontrivial gapped phase ( $\delta z = 0.0$ )

```
In[ ]:= ListPlot[Transpose[Sort /@ EvalData], Joined → True]
      リスト…  転置  並べ替え  点の結合  真
Out[ ]:=
```



The helical edge states survive even in the presence of a time-reversal-symmetry-preserving perturbation.

```
In[ ]:= Sort[Flatten[EvalData]]
      並…  平滑化
Out[ ]:=
```

```
{-2.52909, -2.52909, -2.52862, -2.52862, -2.5277, -2.5277, -2.52722, -2.52722, -2.52683, -2.52683,
-2.52671, -2.52671, -2.52652, -2.52652, ... 11972 ..., 2.52652, 2.52652, 2.52671, 2.52671,
2.52683, 2.52683, 2.52722, 2.52722, 2.5277, 2.5277, 2.52862, 2.52862, 2.52909, 2.52909}
```

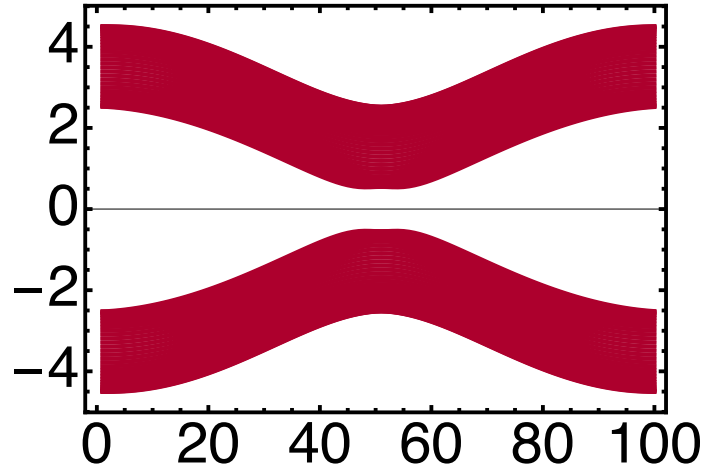
完全な式は使えません (もとのメモリサイズ : 96.2 kB)



Kramers degeneracy is respected.

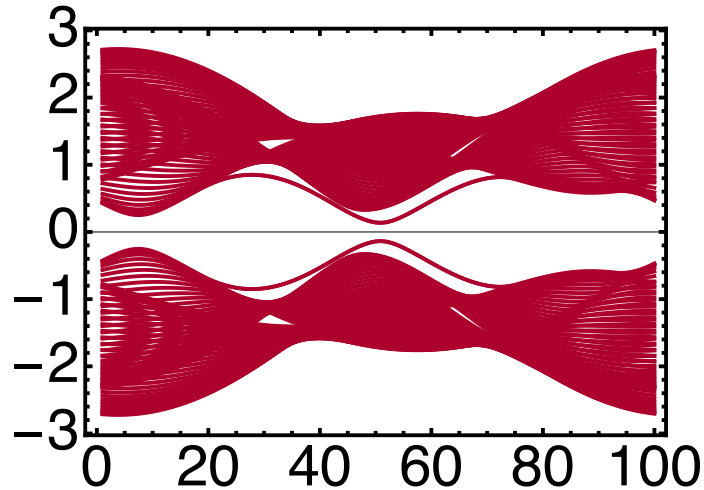
$m = 2.5$  &  $\delta y = 0.3$  : topologically trivial gapped phase ( $\delta z = 0.0$ )

```
In[*]:= ListPlot[Transpose[Sort /@ EvalData], Joined -> True]
Out[*]=
```



$m = 0.5$  &  $\delta y = 0.3$  : topologically nontrivial gapped phase with a time-reversal-symmetry-breaking perturbation ( $\delta z = 0.2$ )

```
In[*]:= ListPlot[Transpose[Sort /@ EvalData], Joined -> True]
Out[*]=
```



The helical edge states are gapped owing to a time-reversal-symmetry-breaking perturbation, even in the presence of a bulk energy gap.

```
In[*]:= Sort[Flatten[EvalData]]
Out[*]=
```

```
{-2.72337, -2.72267, -2.72159, -2.71947, -2.71731, -2.71488, -2.71415, -2.71378, -2.71313, -2.71095,
-2.71055, -2.70891, -2.70561, -2.70528, ... 11972 ..., 2.70528, 2.70561, 2.70891, 2.71055,
2.71095, 2.71313, 2.71378, 2.71415, 2.71488, 2.71731, 2.71947, 2.72159, 2.72267, 2.72337}
```

完全な式は使えません (もとのメモリサイズ: 96.2 kB)



Kramers degeneracy is violated.