Aharonov-Bohm effect[*1]

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28th October 2024

We investigate a charged quantum particle in an electromagnetic potential,

$$H = \frac{\left(\boldsymbol{p} - q\boldsymbol{A}\left(\boldsymbol{r}\right)\right)^2}{2m} + V\left(\boldsymbol{r}\right), \tag{1}$$

where $\boldsymbol{p} = -\mathrm{i}\hbar\partial_{\boldsymbol{r}}$ represents a momentum, m and q respectively denote the particles' mass and charge, and $\boldsymbol{A}$ and V vector and scalar potentials. The Schrödinger equation reads

$$H\left|\psi\right\rangle = E\left|\psi\right\rangle, \tag{2}$$

where E is eigenenergy, and $\left|\psi\right\rangle$ the corresponding eigenfunction. We solve this from the same system without the vector potential $\boldsymbol{A}$,

$$H_0\left|\psi_0\right\rangle = E\left|\psi_0\right\rangle, \quad H_0 \coloneqq \frac{\boldsymbol{p}^2}{2m} + V\left(\boldsymbol{r}\right). \tag{3}$$

From straightforward calculations, we have

$$\left|\psi\right\rangle = \exp\left[\frac{\mathrm{i}q}{\hbar}\int_{\boldsymbol{r}_0}^{\boldsymbol{r}} \boldsymbol{A}\left(\boldsymbol{r}'\right)\cdot d\boldsymbol{r}'\right]\left|\psi_0\right\rangle, \tag{4}$$

where $\boldsymbol{r}_0$ is a reference point in real space.

Now, the Berry connection $\mathcal{A}$ in real space is

$$\mathcal{A} \coloneqq \mathrm{i}\left\langle\psi\middle|\partial_{\boldsymbol{r}}\middle|\psi\right\rangle = -\frac{\left\langle\psi_0\middle|\boldsymbol{p}\middle|\psi_0\right\rangle}{\hbar} - \frac{q}{\hbar}\boldsymbol{A}, \tag{5}$$

reducing to the electromagnetic vector potential $\boldsymbol{A}$. Consequently, the Berry phase ϕ_C acquired over a closed path C in real space is obtained as

$$\phi_C = \oint_C \mathcal{A}\left(\boldsymbol{r}\right)\cdot d\boldsymbol{r} = -\frac{q}{\hbar}\oint_C \boldsymbol{A}\left(\boldsymbol{r}\right)\cdot d\boldsymbol{r} = -\frac{q}{\hbar}\Phi, \tag{6}$$

where Φ is the magnetic flux enclosed by C.

[*1] Y. Aharonov and D. Bohm, *Significance of Electromagnetic Potentials in the Quantum Theory*, Phys. Rev. **115**, 485 (1959).